J J Lash

LED-440-3

LEM MISSION SIMULATOR (LMS)

MATH MODEL

VEHICLE DESIGN INTEGRATION

Vehicle Design Integration material is being provided as a continuing effort under separate cover and shall no longer be referenced in the Math Model. Vehicle Design Material provided to date in the Math Model shall be considered as reference material only except for the Level II schematic.

The Vehicle Design Integration schematic has been added to the introduction to the Math Model.

VEHICLE DESIGN
INTEGRATION

LED-440-3

LEM MISSION SIMULATOR (LMS)

MATH MODEL

TRUE MOTION EQUATIONS

Table of Contents

Paragraph

Part I LEM Vehicle

Not applicable

Part II LMS Data

1. Equation of Motion
2. Primary Guidance and Navigation

TRUE MOTION EQUATIONS

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True Motion Equations

Part II

The True Motion Equations are subdivided into two groups. Section 1 contains data on the Equations of Motion, and Section 2 contains data on the Primary Guidance and Navigation Equations. Each section is divided into four subparagraphs these are: Symbol List; Equations; Equation Documentation, and Intersystem Requirement. The paragraph on Equation Documentation describes the assumptions, simplifications, and methods used to derive the equations appearing in the previous subparagraph. The equations appear on sheets AA through R. Sheets AA through K pertain to the Equation of Motions. Sheets K contains the associated illustrations. Sheets L through R pertain to the Primary Guidance and Navigation equations. Sheets R contain the associated illustrations. The index shown below list the various True Motion Equations.

Section 1. Equation of Motion	Designation
Level II Flow Diagram (2 Sheets)	AA
Translational Equations - M Frame (2 Sheets)	A
LEM-CSM Relative Equations of Motion, and CSM Trajectory	B
Rotational Equations of Motion	C
General Transformations (2 Sheets)	D
Ephermeris	E
Rendezvous Radar	F
LEM Lunar Landing Radar (2 Sheets)	G
LEM-Earth, LEM-CSM Communication Requirements during Lunar and Earth Operations ([illegible] Sheets)	H
Weights and Balance	I
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Part II (Cont)

Section 2. Primary Guidance and Navigation	Designation
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TRUE MOTION EQUATIONS

Part II LMS Data

Section 1. Equation of Motion

1. Symbol Definition
2. Equations
3. Equation Documentation
4. Intersystem Requirements
   a. Boolean Assignments
   b. Continuous Data

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TRUE MOTION EQUATIONS

Part II LMS Data

Section 1. Equation of Motion

1. Symbol Definition

LMS SYMBOL DEFINITIONS

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Part II, Section 1-1
True Motion Equations
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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
a_o	Semi-major axis of CSM orbit.	ft.	5.7×10^6 to $7.\boxed{0} \times 10^6$ 20.9×10^6 to 24.5×10^6	Lunar orbit Earth orbit
a_{pq}, b_{pq}, c_{pq}	Celestial sphere gimbal angle drives.	deg.	0 to 360	Input to EVDE.
a_{ij}	Transformation matrix from inertial M-Frame to selenographic S-Frame.	-	± 1	All direction cosine elements can vary from -1 to +1.
a_{ij}^*	Constant transformation matrix from inertial M-frame to selenographic S-frame computed at some epoch t*.	-	± 1	Constant, once t* is specified.
a_k, b_k, c_k $k = \boxed{1}, 2, 3, 4$	Direction cosines between X_B, Y_B, Z_B body axis and landing radar beam directions	-	± 1	-
A, B, C	Lunar inertia constants: $\frac{I_C - I_A}{I_C}$, $\frac{I_C - I_B}{I_C}$, $\frac{M\ I_C}{2M_m}$	-	$621.1358 \times 10^{-6} = A$ $207.0881 \times 10^{-6} = B$ 1.1263979×10^{27} ft^5/SEC^2 = C	Input constants
A_{LS}	Rendezvous radar azimuth line of sight.	deg.	0 to 360	A_{LS}-Input to RRMM
$A_{x/v}$, $A_{y/v}$, $A_{z/v}$	Aerodynamic drag perturbation components (LEM or CSM).	ft/sec^2	$\pm 6 \times 10^{-5}$	-

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LMS SYMBOL DEFINITIONS

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
A_{Aw_i}, E_{Aw_i} w = l or c	LEM or CSM VHF antenna direction cosines with respect to LEM or CSM body axes.	deg.	-	Input constants.
c_{ij_n}	Transformation matrix from inertial M or E-Frame to ideal IMU R-Frame	--	± 1	Constant matrix.
$c_{1_{ni}}$, $c_{2_{ni}}$, $c_{3_{ni}}$	CSM VHF antenna direction cosines with respect to n-frame.	--	± 1	Supplied by AMS during integrated mode.
d	Mean solar days from January 1.0 1950 to date.	days	(6 to 9) x 10^3	Not required if included in JPL tapes
D_{Si} i = 1,2,3	Landing radar doppler velocity signals.	ft/sec	0 to 500	Input to LRMM.
D*	Integer mean solar days from beginning of launch year to problem start.	days	0 - 365	Input constant

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
e^2	Earth flattening equivalent.	-	0.006693219	Input constant.
$e_1; e_2; e_3; e_4$	Quaternions	-	± 1	-
$(E-E_o)$	Change in CSM eccentric anomaly.	rad.	0 to 2π	Iterate for this parameter.
E_{LS}	Rendezvous radar elevation line of sight.	deg.	0 to 360	E_{LS}-Input from RRMM
f"	Earth flattening parameter	-	$\frac{1}{298.30}$	Input constant
$(F_X; F_Y; F_Z)_B$	Total external force components along LEM body axes.	lbs.	$\pm 12 \times 10^3$	-
g_{ij_n}	Transformation matrix from inertial E or M-frame to LEM body B-frame.	-	± 1	-
g_{ij_c}	Transformation matrix from inertial M or E-frame to CSM body frame.	-	± 1	Supplied by AMS during integrated mode.

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LMS SYMBOL DEFINITIONS

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$g_{\odot}$	Mean longitude of the Sun.	deg.	0 to 360	Required for physical libration.
GHA	Greenwich hour angle	deg.	0 to 360	
g_E	Earth Gravity Constant	ft/sec^2	32.17405	
$h_{ij_{pq}}$	Transformation matrix from LEM-body axis to window or telescope axes.	--	$+\pm 1$	Constant matrix.
$h_{M/L}$	Altitude of LEM CG above lunar surface.	ft.	0 to 6×10^5	Lunar altitude measured with respect to reference spherical surface.
$h_{M/LR}$	Altitude of LEM landing radar above lunar surface.	ft.	0 to 6×10^5	Lunar altitude measured with respect to reference spherical surface.
h_{DE}	Altitude of design eye reference point above lunar surface.	ft.	0 to 3×10^4	Input to Landing and Ascent Image Generator
h	Altitude above spheroidal surface.	ft.	6×10^5 to 3×10^6	Earth orbits.

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
H	Hours (UT) from Greenwich midnight to problem start.	hrs.	0 to 24	-
$H_{n/C}$	Total CSM angular momentum	ft^2/sec	30×10^9	-
H_x; H_y; H_z	Component CSM angular momentum	ft^2/sec	30×10^9	-
i_f	Inclination of MEP film strip relative to lunar equator.	deg.	0 ± 90°	Input constant
I	Hayn's inclination constant of lunar equator to ecliptic.	deg.	1.535	Input constant
I_x; I_y; I_z	Moments of inertia with respect to body B-axes.	$slug\text{-}ft^2$	2000 to 22,000	Does not include CSM.
I_{xy}; I_{yz}; I_{zx}	Products of inertia with respect to B-body axes.	slug-ft	-100 to 700	-
JD	Julian Date	days	$(2.4 \text{ to } 2.5) \times 10^6$	-

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LMS SYMBOL DEFINITIONS

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True Motion Equations
Equations of Motion

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
J_2	Oblateness constant	-	1.62345×10^{-3}	Input constant
K_o	Normalizing Constant	ft./deg	0.9952557×10^5	
$l_{1n_i}, l_{2n_i}, l_{3n_i}$	LEM-VHF antenna direction cosines withrespect to n-frame.	-	± 1	-
l_1, l_2	Distance from fixed RCS reference point to RCS jets.	ft.	4 to 6	Input constant
$l_{ij_{pq}}$	Transformation matrix from LEM window or telescope axis to M or E frame.	-	± 1	-
L_{ij}	Physical lunar libration matrix.	-	± 1	-
$L_B; M_B; N_B$	Total LEM body torques in X_B, Y_B, Z_B directions, about the instantaneous CG.	Ft-lbs	±40,000	-
$L_{\bar{R}}; M_{\bar{R}}; N_{\bar{R}}$	Reaction control torques about RCS fixed reference points.	ft-lbs	±3000	-
$L_R; M_R; N_R$	Reaction control troques about instantaneous CG	ft-lbs	± 4000	-

LMS SYMBOL DEFINITIONS

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Part II, Section 1-1

True Motion Equations
Equations of Motion

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
L_K; M_K; N_K	Main engine (ascent or descent torques about instantaneous CG)	ft-lbs	± 10,000	--
L_{S_K} ; M_{S_K}	Fuel slosh torques about instantaneous CG.	ft-lbs	± 3000	--
M*, N*	Stage separation torque about instantaneous CG.	ft-lbs	± 12,000	--
M_K	Sum of rigid and slosh masses	slugs	Descent 0 → 300 slugs Ascent 0 → 170 slugs	Main engine math model
m_{R_1}	Total RCS propellant mass remaining (system a or b; 1 = a, 1 = b)	slugs	10	Input from RCS Math Model
m_L	Instantaneous total LEM mass	slugs	250 → 1100 slugs	--
m_I	Total dry mass of ascent stage	slugs	200	Input constant
m_{II}	Total dry mass of descent stage	slugs	150	Input constant

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$m_{s_{Kj}}$	Main engine fuel or oxidizer slosh mass in jth tank.	slugs		See A-46a, A-47a
$m_{r_{Dj}}$	Descent engine fuel or oxidizer rigid mass in jth tank.	slugs		See A-46a, A-47a
n_{ij}	Transformation matrix from LEM window or telescope axes to mean ecliptic axes of date.	-	± 1	-
N	Leap year integer correction for computing Julian date.	days	4-6	-
P_{ij}	Transformation from rendezvous and docking display axes ($\hat{\rho}$) to CSM body axes.	-	± 1	-
p_B; q_B; r_B	LEM body rates about X_B, Y_B, Z_B axes respectively, relative to an inertial system.	rad/sec	$\pm \frac{\pi}{3}$	-
p_X, p_Y, p_Z	Lunar triaxiality acceleration components.	ft/sec^2	$\pm 2 \times 10^{-3}$	-
q_{ij}	Transformation from rendezvous and docking display axes ($\hat{\rho}$) to LEM body axes.	-	± 1	-

LMS SYMBOL DEFINITIONS

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$r_{n/V}$ η = M or E ν = L or C	Distance between LEM or CSM CG and Moon or Earth.	ft	6×10^6 24×10^6	—
$r'_{B/L}$	Distance measured from LEM landing radar to Moon center	ft	6×10^6	—
r_{E_m}	Distance from center of Earth to Center of Moon	ft	13×10^8	
R_{K_T}	Ascent or descent tank radius	ft	$R_{AT} = 2.04$ $R_{D_T} = 2.12$	Input constant
$R\ A_E$	Right ascension of Earth take off site at launch	deg.	0 to 360	Input constant
$(RA)_{\odot}$	Right ascension of Sun measured at problem start.	deg.	0 to 360	—

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LMS SYMBOL DEFINITIONS

SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
R_M	Mean radius of moon	ft	5.702395×10^6	Input constant
R_E	Mean equatorial radius of earth	ft	20.92573818×10^6	Input constant
R_k $k = 1,2,3,4$	Slant range along each landing radar beam from LEM to lunar surface.	ft	0 to 7.5×10^5	Input to LRMM
R_{LM}	Design lunar radius based on Landmass Simulator datum reference.	ft	5.7×10^6	Input constant. The value depends on the intended landing or takeoff site.
$\overline{S}_z$	Component fuel and oxidizer slosh force in LEM body coordinates.	lbs	± 400	-
S_x^*	Stage separation force	lbs	$(3 \text{ to } 7) \times 10^3$	
S_V	Reference Area	ft^2	-	Input constant
t	Problem time	sec	-	-

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
t^*	Time measured from problem start which specifies the position of the IMU vertical X_R direction (landing site at landing or takeoff at takeoff)	sec	--	Input constant
T_o	Julian centuries measured from Jan. 1.0 1950 to problem start	J. cent's	--	Required for JPL tapes
T^*	Julian centuries measured from Jan. 1.0 1950 to t^*	J. Cent's	--	--
T_u $u = 1, 2 \ldots 16$	RCS thrust	lbs	0 to 100	Input from RCSMM
T_K	Main engine thrust (ascent or descent)	lbs	0 to 3,500 0 to 10,500	Input from MEMM
$T_{X_{BR}}; T_{Y_{BR}}; T_{Z_{BR}}$	RCS thrust components along body axes.	lbs	0 to 400	Input from RCSMM
$T_{X_{BK}}; T_{Y_{BK}}; T_{Z_{BK}}$	Main engine thrust components along body axes.	lbs	11,000	Input from MEMM
$V_{R/V}$	Velocity of LEM or CSM relative to atmosphere.	ft/sec	0 $\longrightarrow$ 28×10^3	--

SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$\ddot{W}_s$	Component slosh acceleration parameter in Z_B body coordinates	ft/sec^2	± 5	--
$V_{n/C}$	CSM velocity in inertial M- or E-frame	ft/sec	5000 → 7000 24000 → 30000	Moon Earth
X_{LM}; Y_{LM}	Displacement of subsatellite point with respect to origin of Landing Table Model or Landmass Simulator	ft		Input to EVDE and Landmass Simulator
x^n_{pq}, y^n_{pq}	Coordinates of Moon or Earth occulter in window or telescope axes	ft	0 - 150°	--
$X_{n/L}$; $Y_{n/L}$; $Z_{n/L}$	LEM position coordinates in inertial M- or E-frame	ft	5.7 to 6.5 x 10^6 24 to 25.5 x 10^6	Moon Earth
$X_{B/L}$; $Y_{B/L}$; $Z_{B/L}$	LEM position coordinates measured in body frame	ft	"	"
$X_{n/c}$; $Y_{n/c}$; $Z_{n/c}$	CSM position coordinates in inertial M- or E-frame	ft	"	"

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$\dot{X}_{B/S};\dot{Y}_{B/S};\dot{Z}_{B/S}$	Component velocities along LEM body axes with respect to lunar surface.	ft/sec	0 to 6000	—
$\dot{X}_{M/S};\dot{Y}_{M/S};\dot{Z}_{M/S}$	Component velocities of lunar surface measured in M-frame.	ft/sec	0 to 15	—
$X_{S/V};Y_{S/V};Z_{S/V}$	LEM or CSM coordinates in selenographic S-frame.	ft	5.7 to 6.5×10^6	—
$X_{E/M};Y_{E/M};Z_{E/M}$	Position of Moon in E-frame	ft	15×10^8	Input from JPL tapes
$X_{E/\odot};Y_{E/\odot};Z_{E/\odot}$	Position of Sun in E-frame.	ft	60×10^{10}	Input from JPL tapes
$X_{LAS};Y_{LAS};Z_{LAS}$	Position of shadow in terrain frame		0 to 20,000 ft	
$\dot{X}_{R/n};\dot{Y}_{R/n};\dot{Z}_{R/n}$	Velocity components of vehicle relative to atmosphere	ft/sec	0 to 28×10^3	—
$X_x;Y_y;Z_z$	Elements of precession matrix	---	± 1	Included in JPL tapes
		---		
$\Delta Y4, \Delta Z4$	Radar beam intersection with lunar surface		0 - 130,000ft	

LMS SYMBOL DEFINITIONS

SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
Y	Launch year	---	1969 to 1975	Input constant
α_j	Position of LEM landing radar plate relative to LEM body axes.	deg.	-21° to +54°	Input constants
$\bar{\alpha}, \bar{\beta}, \bar{\gamma}$	Distance from LEM CG to local CG of any particular item along X_B, Y_B, Z_B directions, respectively.	ft.	± 20	--
$\alpha_{CG}, \beta_{CG}, \gamma_{CG}$	Distance from fixed body reference axis to instantaneous LEM CG.	ft.	± 20	--
α, β, γ	Distance from fixed body reference axis to local CG of any particular item.	ft.	± 20	--
$\Delta\alpha_{Dj}$	Distance from center of j^{th} descent tank to CG of remaining oxidizer or fuel.	ft.	---	--
$\gamma^{\odot}$	Angle made by the projection of the sun is the $\widehat{\rho}_1, \widehat{\rho}_2$ rendezvous display plane with direction $\widehat{\rho}_1$.	deg.	0 to 360	--

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$\chi_{pq}^{\odot}$	Angle between Sun direction and optical axis line of sight direction.	deg.	0 to 180	---
Γ'	Mean longitude of the lunar perigee	deg.	0 to 360	---
δ_K	Declination of landing or take-off site.	deg.	± 20°	Input constant
δ_f	Angular displacement of sub-satellite relative to MEP film strip center-line.	deg.	± 20°	---
$\delta_{\theta K}, \delta_{\psi K}$	Main engine gimbal angles	deg.	0 to 10	Input from SCMM
$\delta_{S/4}$	Radar beam intersection in film coordinates		0 - 360 deg.	
ϵ	Obliquity of the ecliptic	deg.	23	---

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LMS SYMBOL DEFINITIONS

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
ϵ^*	Tolerance parameter.	deg	0.1	Input constant
ζ_{ii}	LEM-CSM polarization angle measured in a plane normal to the line-of-sight vecotr.	deg	0 to 180	Input to CRMM
θ_f	Position of LEM subsatellite point relative to ascending node of MEP film strip.	deg	0 to 360	-
θ_c, ϕ_c	Earth communication antenna gimbal angles	deg	0 to 360	Input to Communication Math Model
θ_{ok} $k = 1,2,3,4$	The angle between each landing radar beam direction and the local vertical formed by the intersection of each beam direction with the lunar surface	deg	0 to 90	Input to LRMM
θ_{LS}, ϕ_{LS}	Orientation of LEM line-of-sight relative to rendezvous display axes (LEM camera angle drives)	deg	0 to 360	Input to EVDE
θ, ϕ, ψ	LEM Euler angles (ordered rotations given by θ, then ψ then ϕ)	deg	0 to 360	-
θ_c, ϕ_c, ψ_c	CSM Euler angles (ordered rotations given by ψ_c, then θ_c, then ϕ_c)	deg	0 to 360	Supplied by AMS or Instructor

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$(\theta_G)_{CSM}$; $(\phi_G)_{CSM}$ $(\psi_G)_{CSM}$	1/20 or 1/80 scale CSM CSM docking model gimbal angle rotations	deg	0 to 360	Input to EVDE
θ_{pq}, ϕ_{pq}, ψ_{pq}	Orientation of window or telescope axis relative to LEM body axes.	deg	-	Input constants
$\theta_{S/4}$	Radar beam intersection in film coordinates	deg	0 to 90°	-
				-
$\lambda_{S/V}$	Selenographic longitude of either LEM or CSM	deg	0 to 360	-
λ_{SK}	Selenographic longitude of either the landing site or take-off site	deg	0 to 360	Input constant
Λ_i $i = 1, 2, 3$	Angle between landing radar beams.	deg		Input constants
$\Delta\Lambda_i$	Angular error between landing radar beams	deg		Input constants

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
μ_K K=1, 2, 3, 4	Angle between slant range vector and local vertical	deg	0 to 90	-
μ_n	Central force gravitational constant of moon or earth	$\frac{ft^3}{sec^2}$	$1.73139972 \times 10^{14}$ $1.40765391 \times 10^{16}$	Input constants
μ^*	Semi angle of moon subtended by the LEM vehicle	deg	0 to 90	-
ξ_{V_i}	LEM or CSM VHF antenna angles with respect to the line-of-sight vector	deg	0 to 180	Input to Communication Math Model
ξ'_i	LEM spiral antenna angles with respect to line-of-sight vector	deg	0 to 180	Input to Communication Math Model
ξ_K	Damping ratio of descent engine or ascent engine fuel or oxidizer	-	-	Input constant
ξ_k k=1,2,3,4	Angle between landing radar beams	deg	-	Input constant
$\Delta \xi_k$	Angular error in direction of landing radar beams	deg	-	Input constant

LMS SYMBOL DEFINITIONS

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Part II, Section 1-1
True Motion Equations
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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
ρ_{LS}	Line-of-sight distance between LEM-CG and CSM-CG	ft	24×10^5	—
ρ'_{LS}	Line-of-sight distance measured from design eye to CSM-CG	ft	24×10^5	—
ρ_X, ρ_Y, ρ_Z	Component relative distances of CSM WRT LEM measured in E- or M-frame directions	ft	$\pm 24 \times 10^5$	—
$\rho_{X_B}, \rho_{Y_B}, \rho_{Z_B},$	Component relative distances of CSM WRT LEM measurec in body B-frame directions	ft	$\pm 24 \times 10^5$	—
$\rho_{pq_{max}}$	Maximum occulter position angle	deg	150°	Input constants
ρ_V	Atmospheric density	$slugs/ft^3$		Table look-up
$\rho_{XLA}, \rho_{YLA}, \rho_{ZLA}$	Position of Shadow in LEM body frame	ft	0 - 20,000ft	—
$\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3$	Rendezvous and docking model reference axes		----	—
$\rho_1^{\odot}, \rho_2^{\odot}, \rho_3^{\odot}$	Coordinates of sun in $\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3$ reference frame	—	± 1	—

LMS SYMBOL DEFINITIONS

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Part II, Section 1 -1
True Motion Equations
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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
σ_c	Central angle between LEM-moon radius vector and CSM-moon radius vector	deg	0 to 180	—
σ_{Ei}	Central angle measured at moon between the LEM-moon radius vector, and the moon i^{th} earth station radius vector	deg	0 to 180	—
$\sigma^{\odot}$	Angle between $\hat{\rho}_3$ axis and sun's direction	deg	0 to 180	—
$\sigma^{\odot}_{min,\ max}$	Fixed angles measured between $\hat{\rho}_3$ and the suns direction	deg	—	Input constant
$\sigma^{\odot}_{M,\ E}$	Angle between either $r_{M/C}$ or $r_{E/C}$ and sun's direction	deg	0 to 180	Input to EVDE
σ_{*pq}	Angle between window or t telescope optical axis and LEM local vertical		—	—
$\phi_{S/V}$	Selenographic latitude of either the LEM or CSM vehicle	deg	0 to ± 90	—

SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
ϕ_{SK}	Selenographic latitude of either the lunar landing site or take-off site	deg.	$\pm 90°$	Input constant
ϕ_k	Angular displacement of the tank coordinate system about the X_B LEM axis	deg.	0 to 360	———
ϕ^*_{pq}	Angle which measures roll about optical line-of-sight	deg.	0 to 360	Input to EVDE
ψ^*_{pq}	Angle measured between the projection of window or telescope optical axes on the lunar surface and the direction of the MEP film strip centerline.	deg.	0 to 360	Input to EVDE
Ω	Longitude of lunar orbit ascending mode	deg.	0 to 360	———
$\dot{\Omega}$	Nodal regression rate of CSM (Earth orbit)	deg/sec	6×10^{-5}	———
Ω_f	Right ascension of ascending node of MEP film strip centerline measured in selenographic or geographic coordinates.	deg.	0 to 360	Input constant

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Equations of Motion

LMS SYMBOL DEFINITIONS

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SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
$\omega_{x_s}; \omega_{y_s}; \omega_{z_s}$	Total angular velocity of the moon in selenographic coordinates	rad/sec	3×10^{-6}	---
ω_E	Earth rotation rate	deg/sec	4×10^{-3}	---
ω_n	Natural frequency (fuel slosh)	rad/sec	---	See A-46a, A-47a
☾	Mean longitude of moon measured in ecliptic from mean equinox of date to the mean ascending mode of the lunar orbit, and then along the orbit	deg	0 to 360	---

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True Motion Equations
Equation of Motion

LMS SYMBOL DEFINITIONS

Date 11/65

SYMBOL	DEFINITION	UNITS	RANGE	REMARKS
Mh_i	Selenographic to Film Frame	-	± 1	
Mgh	Film to Terrain Frame	-	± 1	
$\sqrt{g_K}'$	Inertial to Terrain Frame	-	± 1	
$\sqrt{g_{kpq}}$	Terrain to optical Frame	-	± 1	
$\sqrt{g_k}^*$	Terrain to body Frame	-	± 1	
$\theta^{\odot}, \psi^{\odot}$	Elevation and Azimuth of sun WRT landing area frame	deg	0-360°	
$\rho_{pq}^{n}, \theta_{pq}^{n}$	Occulter position angles	deg	0-360°	
$\alpha_{pq}^{\odot}$	Sun shafting angle	deg	0-360°	

SUBSCRIPTS

Subscript Symbol	Definition
a	RCS system a.
A	Ascent engine.
b	RCS system b.
B	LEM body axes B-frame.
C	CSM, also communications antenna.
CG	Center of gravity.
D	Descent engine.
DE	Design eye.
e	Expendables.
E	Earth, also geocentric mean equinox reference system.
f	MEP film strip or landing table display.
G	Earth ground station, also Earth fixed references.
H	Local horizon, local vertical reference system.
ib	Inboard axis.
K	Either A (Ascent Engine) or D (Descent Engine).
l	RCS system a or b.
L	LEM
LR	Landing radar antenna.
LS	Line-of-sight.
LM	Land Mass Simulator.
M	Moon; also selenocentric mean equinox reference system.
N	Nozzle
n	Either E-frame or M-frame.
O	Initial condition.

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SUBSCRIPTS (Cont'd)

Subscript Symbol	Definition
ob	Outboard axis.
r	Roll about tracking line; also denotes rigid portion of fuel or oxidizer mass.
p	Window (W) or telescope (T).
q	Right (r), left (l) or center (c) window of telescope.
R	RCS jets, also IMU reference system, also relative.
$\overline{R}$	Reference point of RCS jets.
RR	Rendezvous radar.
S	Selenographic reference system; also refers to fuel slosh.
T	Table-top axes (Land Mass Simulator).
V	Vehicle, either L (LEM) or C (CSM).
X; Y; Z	With respect to X, Y, Z directions.
I	With respect to dry weight of LEM vehicle ascent stage.
II	With respect to dry weight of LEM vehicle descent stage.
O	Sun

TRUE MOTION EQUATIONS

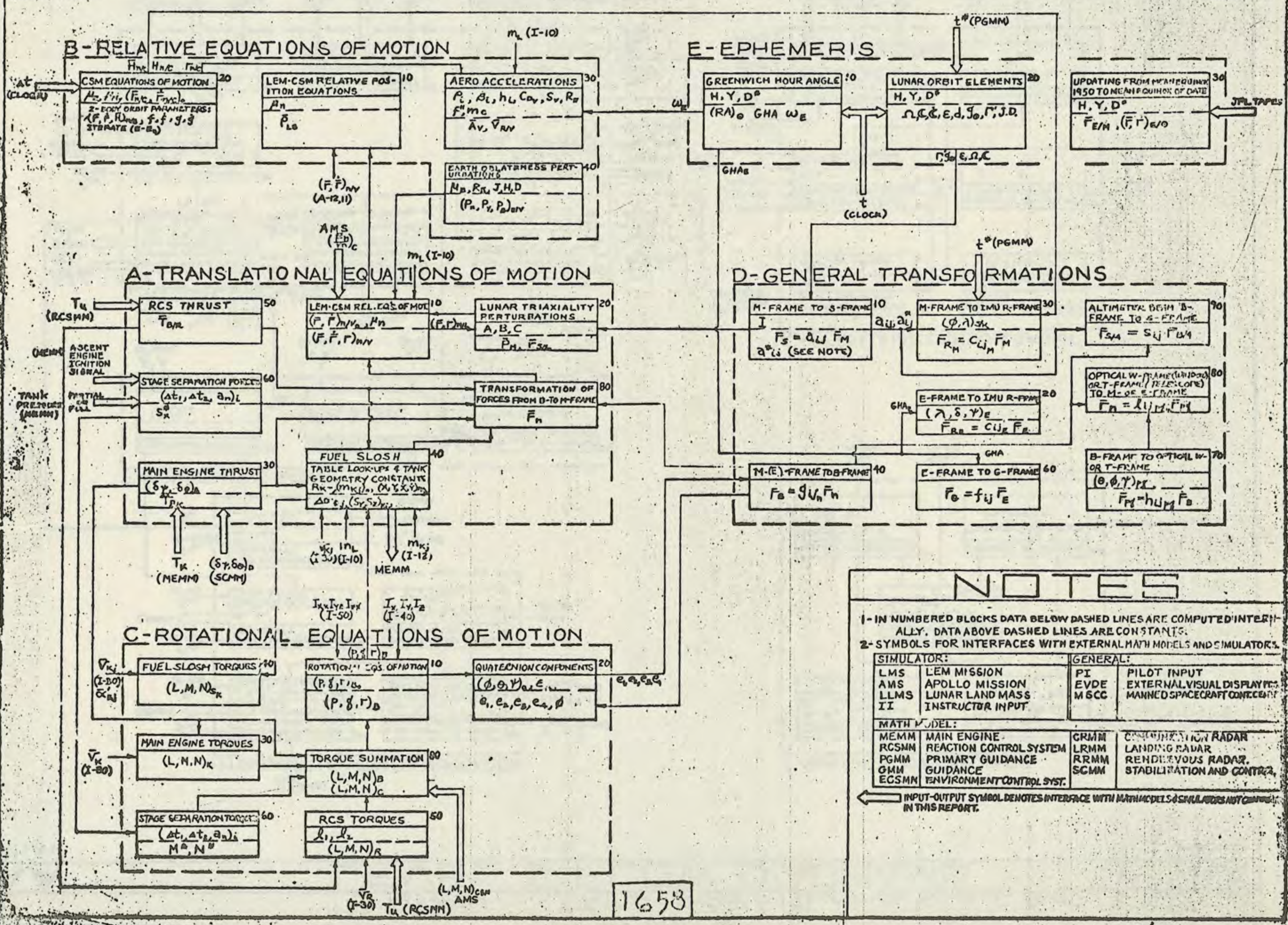
Part II IMS Data

Section 1. Equation of Motion

2. Equations

LEVEL II FLOW DIAGRAMS

AA SHEET 1 OF 2 AA SHEET 1 OF 2



NOTES

1- IN NUMBERED BLOCKS DATA BELOW DASHED LINES ARE COMPUTED INTERNALLY. DATA ABOVE DASHED LINES ARE CONSTANTS.

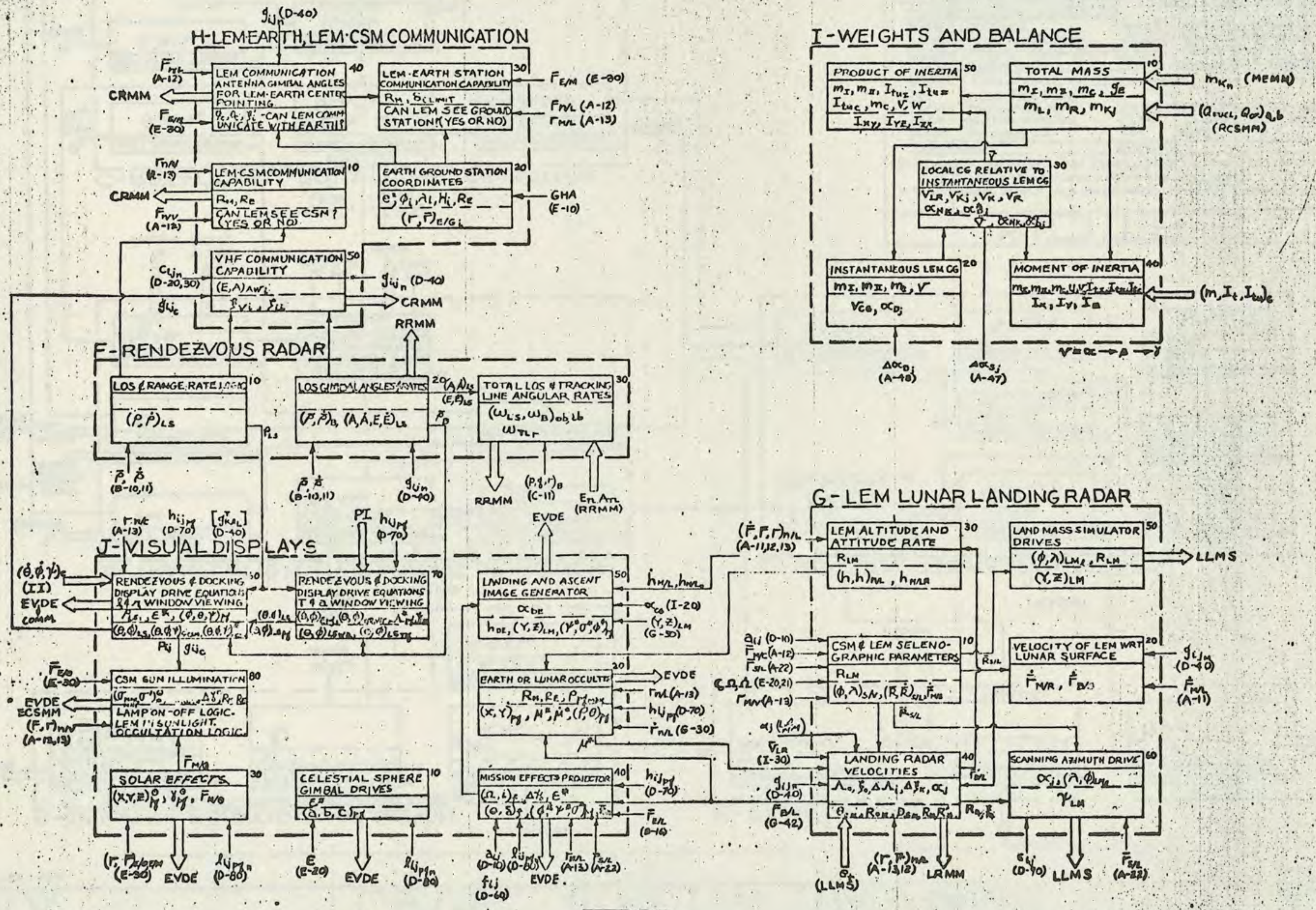
2- SYMBOLS FOR INTERFACES WITH EXTERNAL MATH MODELS AND SIMULATORS.

SIMULATOR:		GENERAL:	
LMS	LEM MISSION	PI	PILOT INPUT
AMS	APOLLO MISSION	EVDE	EXTERNAL VISUAL DISPLAY [illegible]
LLMS	LUNAR LAND MASS	MSCC	MANNED SPACECRAFT CONT. CENT.
II	INSTRUCTOR INPUT		
MATH MODEL:			
MEMM	MAIN ENGINE	CRMM	COMMUNICATION RADAR
RCSMM	REACTION CONTROL SYSTEM	LRMM	LANDING RADAR
PGMM	PRIMARY GUIDANCE	RRMM	RENDEZVOUS RADAR.
GMM	GUIDANCE	SCMM	STABILIZATION AND CONTROL.
ECSMM	ENVIRONMENT CONTROL SYST.		

⇦ INPUT-OUTPUT SYMBOL DENOTES INTERFACE WITH MATH MODELS & SIMULATORS NOT CONTAINED IN THIS REPORT.

AA LEVEL II FLOW DIAGRAMS AA

SHEET 2 OF 2



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A- TRANSLATIONAL EQUATIONS, M-FRAME A

SHEET 1 OF 2

CONSTANTS & INITIAL CONDITIONS 01

$\mu_M = 1.73139972 \times 10^{14}$ FT^3/SEC^2	$(X_{n/v}, Y_{n/v}, Z_{n/v})_0$
$R_M = 5.702395 \times 10^6$ FT.	$(\dot{X}_{n/v}, \dot{Y}_{n/v}, \dot{Z}_{n/v})_0$
$A = 6.[illegible] \times 10^{-6}$ 621.1358	$\delta_{\psi_A}, \delta_{\theta_A}$
$B = 202[illegible]70 \times 10^{-6}$ 207,080	$\delta_{\psi_D}, \delta_{\theta_D}$ FROM CONTROL MATH MODEL
$C = 2.0[illegible]48 \times 10^{27}$ FT^5/SEC^2 1.1263979	R_{AT}, R_{DT}

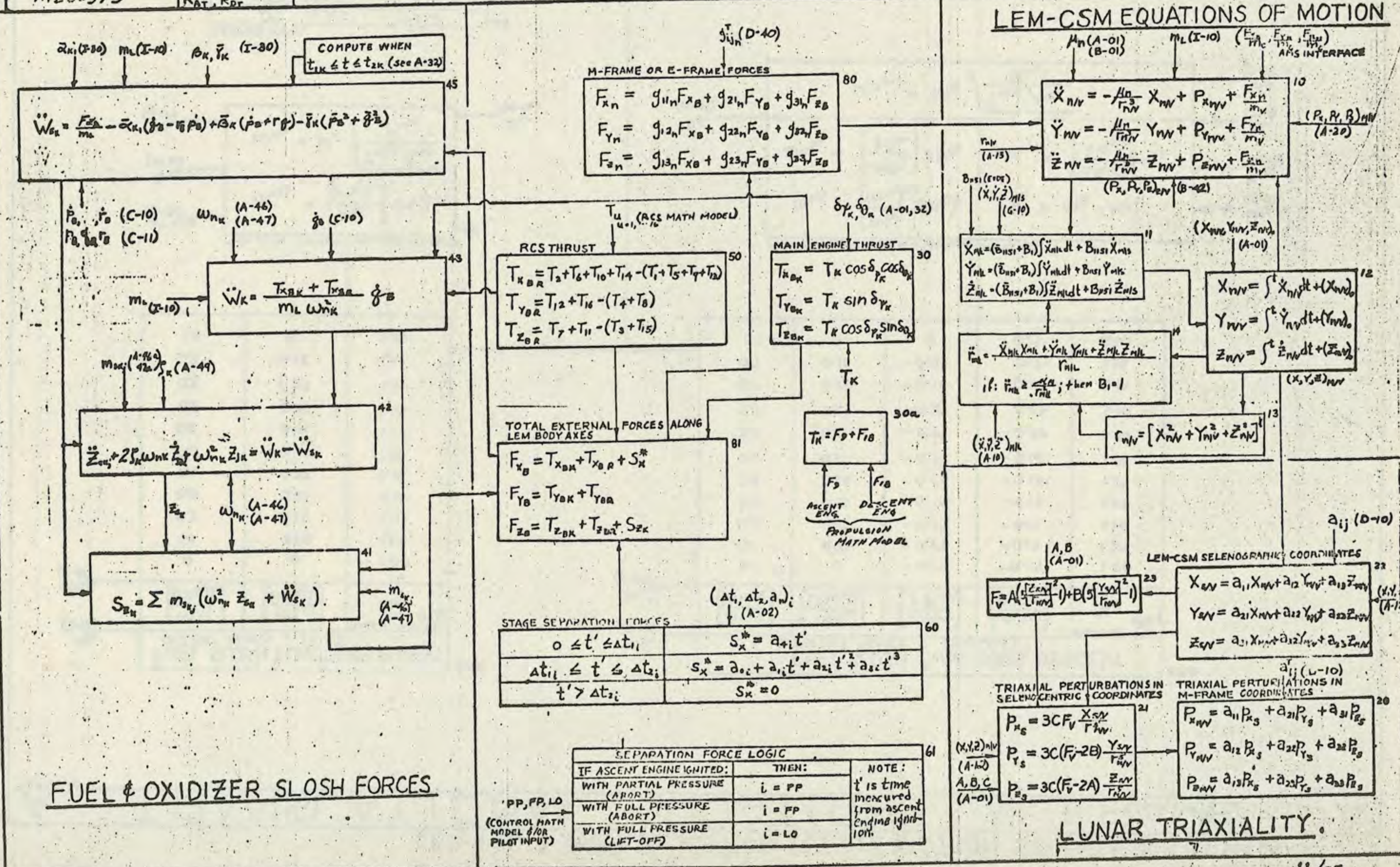
ASCENT-DESCENT ENGINE LOGIC 32

IF:	THEN:	
ASCENT ENGINE STARTED	K = A j = 1,2	$\delta_{\psi_A}, \delta_{\theta_A}$ = CONSTANT THRUST MISALIGNMENT ANGLES t_{1A} = TIME ASCENT ENGINE IS ACTIVATED t_{2A} = TIME ASCENT ENGINE IS DEACTIVATED
DESCENT ENGINE STARTED	K = D j = 1,2,3,4	$\delta_{\psi_D}, \delta_{\theta_D}$ = SUPPLIED BY CONTROL MATH MODEL OR PILOT INPUT t_{1D} = TIME DESCENT ENGINE IS ACTIVATED t_{2D} = TIME DESCENT ENGINE IS DEACTIVATED

t_{i_K} SUPPLIED BY GUIDANCE MATH MODEL &/OR CONTROL MATH MODEL OR PILOT INPUT

CONSTANTS & INITIAL CONDITIONS 02

	SUBSCRIPT i		
	PP	FP	LO
Δt_{1i}	~~0.110~~	0.035	0.100
Δt_{2i}	~~0.800~~	0.600	0.800
a_{0i}	~~3948~~	6,000	6,982
a_{1i}	~~-17,[illegible]70~~	-22,600	-28,000
a_{2i}	~~30,000~~	24,000	42,570
a_{3i}	~~-18,460~~	-5,000	-23,100
a_{4i}	~~22,250~~	50,000	45,850



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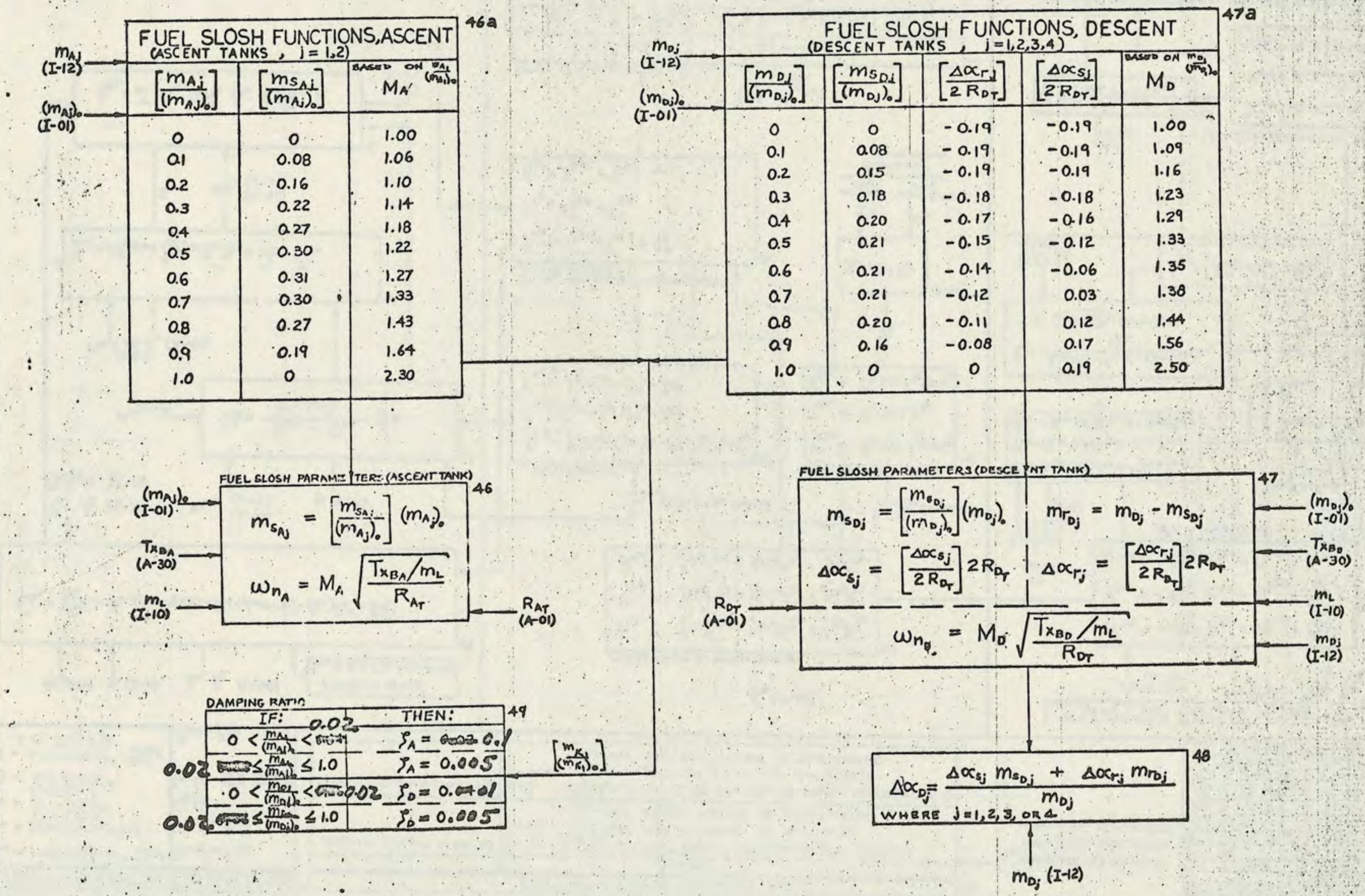
A TRANSLATIONAL EQUATIONS, M-FRAME A

SHEET 2 OF 2



FUEL SLOSH FUNCTIONS, ASCENT
(ASCENT TANKS, j = 1,2)

$\left[\frac{m_{Aj}}{(m_{Aj})_0}\right]$	$\left[\frac{m_{SAj}}{(m_{Aj})_0}\right]$	BASED ON $\frac{m_{Aj}}{(m_{Aj})_0}$ M_A
0	0	1.00
0.1	0.08	1.06
0.2	0.16	1.10
0.3	0.22	1.14
0.4	0.27	1.18
0.5	0.30	1.22
0.6	0.31	1.27
0.7	0.30	1.33
0.8	0.27	1.43
0.9	0.19	1.64
1.0	0	2.30

FUEL SLOSH FUNCTIONS, DESCENT
(DESCENT TANKS, j = 1,2,3,4)

$\left[\frac{m_{Dj}}{(m_{Dj})_0}\right]$	$\left[\frac{m_{SDj}}{(m_{Dj})_0}\right]$	$\left[\frac{\Delta\alpha_{rj}}{2R_{DT}}\right]$	$\left[\frac{\Delta\alpha_{sj}}{2R_{DT}}\right]$	BASED ON $\frac{m_{Dj}}{(m_{Dj})_0}$ M_D
0	0	-0.19	-0.19	1.00
0.1	0.08	-0.19	-0.19	1.09
0.2	0.15	-0.19	-0.19	1.16
0.3	0.18	-0.18	-0.18	1.23
0.4	0.20	-0.17	-0.16	1.29
0.5	0.21	-0.15	-0.12	1.33
0.6	0.21	-0.14	-0.06	1.35
0.7	0.21	-0.12	0.03	1.38
0.8	0.20	-0.11	0.12	1.44
0.9	0.16	-0.08	0.17	1.56
1.0	0	0	0.19	2.50

FUEL SLOSH PARAMETERS (ASCENT TANK)

$$m_{s_{Aj}} = \left[\frac{m_{SAj}}{(m_{Aj})_0}\right](m_{Aj})_0$$

$$\omega_{n_A} = M_A\sqrt{\frac{T_{XBA}/m_L}{R_{AT}}}$$

FUEL SLOSH PARAMETERS (DESCENT TANK)

$$m_{SDj} = \left[\frac{m_{SDj}}{(m_{Dj})_0}\right](m_{Dj})_0 \qquad m_{rDj} = m_{Dj} - m_{SDj}$$

$$\Delta\alpha_{sj} = \left[\frac{\Delta\alpha_{sj}}{2R_{DT}}\right]2R_{DT} \qquad \Delta\alpha_{rj} = \left[\frac{\Delta\alpha_{rj}}{2R_{DT}}\right]2R_{DT}$$

$$\omega_{n_D} = M_D\sqrt{\frac{T_{XBD}/m_L}{R_{DT}}}$$

$$\Delta\alpha_{Dj} = \frac{\Delta\alpha_{sj}\, m_{SDj} + \Delta\alpha_{rj}\, m_{rDj}}{m_{Dj}}$$

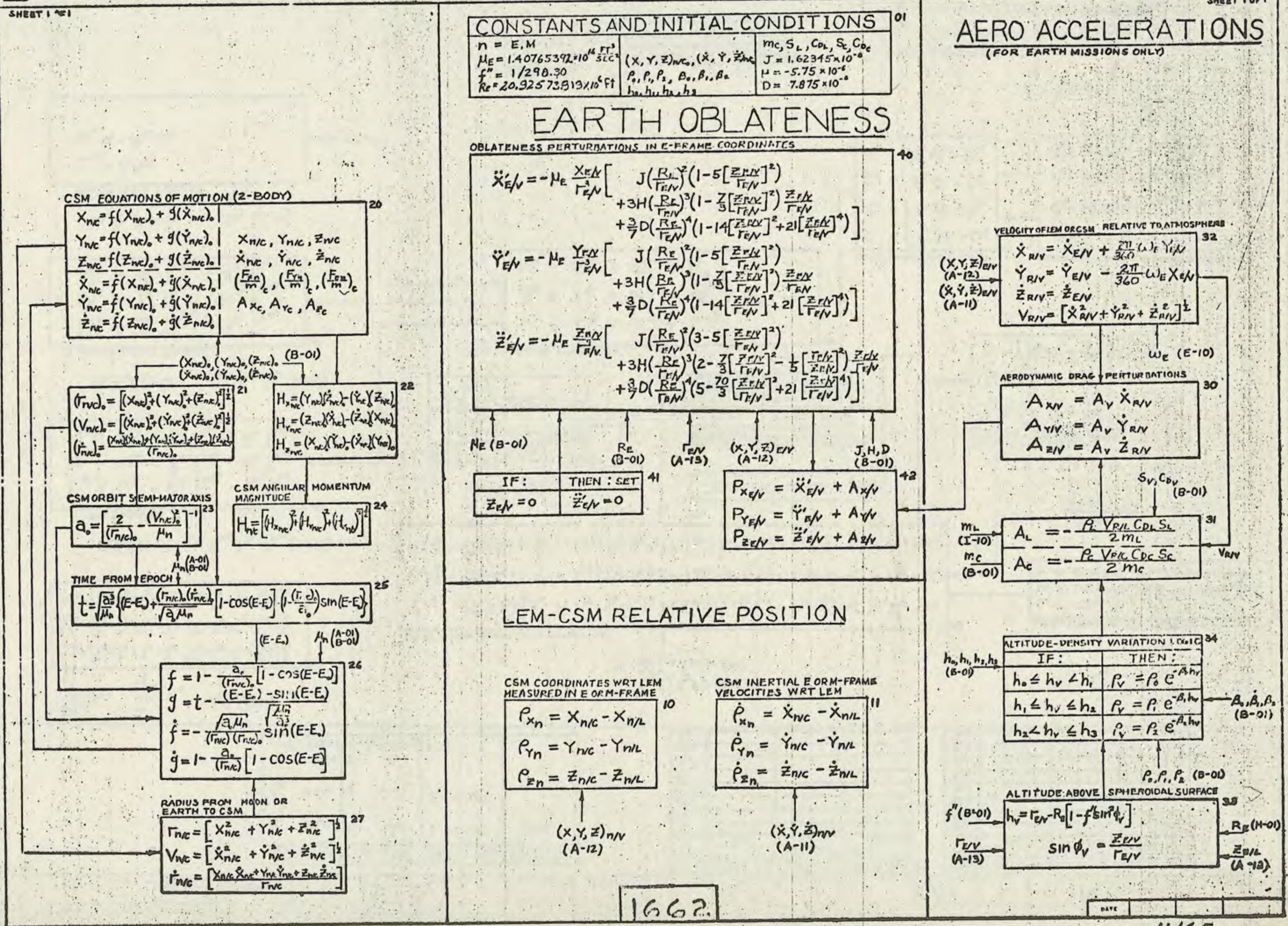
WHERE j = 1, 2, 3, OR 4

DAMPING RATIO

IF:	THEN:
$0 < \frac{m_{Aj}}{(m_{Aj})_0} < 0.02$	$\zeta_A = 0.01$
$0.02 \le \frac{m_{Aj}}{(m_{Aj})_0} \le 1.0$	$\zeta_A = 0.005$
$0 < \frac{m_{Dj}}{(m_{Dj})_0} < 0.02$	$\zeta_D = 0.01$
$0.02 \le \frac{m_{Dj}}{(m_{Dj})_0} \le 1.0$	$\zeta_D = 0.005$

B-LEM-CSM RELATIVE POSITION EQUATIONS & CSM TRAJECTORY-B

SHEET 1 OF 1



CONSTANTS AND INITIAL CONDITIONS 01

$n = E, M$ $\mu_E = 1.40765392\times10^{16}\ \frac{ft^3}{sec^2}$ $f'' = 1/298.30$ $R_E = 20.92573819\times10^6$ ft	$(X,Y,Z)_{n/c_0}, (\dot X,\dot Y,\dot Z)_{n/c}$ $\rho_0, \rho_1, \rho_2, \beta_0, \beta_1, \beta_2$ h_0, h_1, h_2, h_3	$m_c, S_L, C_{D_L}, S_c, C_{D_c}$ $J = 1.62345\times10^{-3}$ $H = -5.75\times10^{-6}$ $D = 7.875\times10^{-6}$

CSM EQUATIONS OF MOTION (2-BODY) 20

$$X_{n/c} = f(X_{n/c})_0 + g(\dot X_{n/c})_0$$
$$Y_{n/c} = f(Y_{n/c})_0 + g(\dot Y_{n/c})_0$$
$$Z_{n/c} = f(Z_{n/c})_0 + g(\dot Z_{n/c})_0$$
$$\dot X_{n/c} = \dot f(X_{n/c})_0 + \dot g(\dot X_{n/c})_0$$
$$\dot Y_{n/c} = \dot f(Y_{n/c})_0 + \dot g(\dot Y_{n/c})_0$$
$$\dot Z_{n/c} = \dot f(Z_{n/c})_0 + \dot g(\dot Z_{n/c})_0$$

$X_{n/c}, Y_{n/c}, Z_{n/c}$

$\dot X_{n/c}, \dot Y_{n/c}, \dot Z_{n/c}$

$\left(\frac{F_{xn}}{m}\right)_c, \left(\frac{F_{Yn}}{m}\right)_c, \left(\frac{F_{Zn}}{m}\right)_c$

$A_{X_c}, A_{Y_c}, A_{Z_c}$

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$$(r_{n/c})_0 = \left[(X_{n/c})_0^2 + (Y_{n/c})_0^2 + (Z_{n/c})_0^2\right]^{\frac{1}{2}}$$
$$(V_{n/c})_0 = \left[(\dot X_{n/c})_0^2 + (\dot Y_{n/c})_0^2 + (\dot Z_{n/c})_0^2\right]^{\frac{1}{2}}$$
$$(\dot r_{n/c})_0 = \frac{(X_{n/c})_0(\dot X_{n/c})_0 + (Y_{n/c})_0(\dot Y_{n/c})_0 + (Z_{n/c})_0(\dot Z_{n/c})_0}{(r_{n/c})_0}$$

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$$H_{x_{n/c}} = (Y_{n/c})_0(\dot Z_{n/c})_0 - (\dot Y_{n/c})_0(Z_{n/c})_0$$
$$H_{Y_{n/c}} = (Z_{n/c})_0(\dot X_{n/c})_0 - (\dot Z_{n/c})_0(X_{n/c})_0$$
$$H_{Z_{n/c}} = (X_{n/c})_0(\dot Y_{n/c})_0 - (\dot X_{n/c})_0(Y_{n/c})_0$$

CSM ORBIT SEMI-MAJOR AXIS 23

$$a_0 = \left[\frac{2}{(r_{n/c})_0} - \frac{(V_{n/c})_0^2}{\mu_n}\right]^{-1}$$

CSM ANGULAR MOMENTUM MAGNITUDE 24

$$H_{n/c} = \left[(H_{x_{n/c}})^2 + (H_{Y_{n/c}})^2 + (H_{Z_{n/c}})^2\right]^{\frac{1}{2}}$$

TIME FROM EPOCH 25

$$t = \sqrt{\frac{a_0^3}{\mu_n}}\left\{(E-E_0) + \frac{(r_{n/c})_0(\dot r_{n/c})_0}{\sqrt{a_0\mu_n}}\left[1-\cos(E-E_0)\right]\left(1-\frac{(r_{n/c})_0}{a_0}\right)\sin(E-E_0)\right\}$$

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$$f = 1 - \frac{a_0}{(r_{n/c})_0}\left[1-\cos(E-E_0)\right]$$
$$g = t - \frac{(E-E_0) - \sin(E-E_0)}{\sqrt{\frac{\mu_n}{a_0^3}}}$$
$$\dot f = -\frac{\sqrt{a_0\mu_n}}{(r_{n/c})(r_{n/c})_0}\sin(E-E_0)$$
$$\dot g = 1 - \frac{a_0}{(r_{n/c})}\left[1-\cos(E-E_0)\right]$$

RADIUS FROM MOON OR EARTH TO CSM 27

$$r_{n/c} = \left[X_{n/c}^2 + Y_{n/c}^2 + Z_{n/c}^2\right]^{\frac{1}{2}}$$
$$V_{n/c} = \left[\dot X_{n/c}^2 + \dot Y_{n/c}^2 + \dot Z_{n/c}^2\right]^{\frac{1}{2}}$$
$$\dot r_{n/c} = \left[\frac{X_{n/c}\dot X_{n/c} + Y_{n/c}\dot Y_{n/c} + Z_{n/c}\dot Z_{n/c}}{r_{n/c}}\right]$$

EARTH OBLATENESS

OBLATENESS PERTURBATIONS IN E-FRAME COORDINATES 40

$$\ddot X'_{E/V} = -\mu_E\frac{X_{E/V}}{r_{E/V}^3}\left[J\left(\frac{R_E}{r_{E/V}}\right)^2\left(1-5\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2\right) + 3H\left(\frac{R_E}{r_{E/V}}\right)^3\left(1-\frac{7}{3}\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2\right)\frac{Z_{E/V}}{r_{E/V}} + \frac{3}{7}D\left(\frac{R_E}{r_{E/V}}\right)^4\left(1-14\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2+21\left[\frac{Z_{E/V}}{r_{E/V}}\right]^4\right)\right]$$

$$\ddot Y'_{E/V} = -\mu_E\frac{Y_{E/V}}{r_{E/V}^3}\left[J\left(\frac{R_E}{r_{E/V}}\right)^2\left(1-5\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2\right) + 3H\left(\frac{R_E}{r_{E/V}}\right)^3\left(1-\frac{7}{3}\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2\right)\frac{Z_{E/V}}{r_{E/V}} + \frac{3}{7}D\left(\frac{R_E}{r_{E/V}}\right)^4\left(1-14\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2+21\left[\frac{Z_{E/V}}{r_{E/V}}\right]^4\right)\right]$$

$$\ddot Z'_{E/V} = -\mu_E\frac{Z_{E/V}}{r_{E/V}^3}\left[J\left(\frac{R_E}{r_{E/V}}\right)^2\left(3-5\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2\right) + 3H\left(\frac{R_E}{r_{E/V}}\right)^3\left(2-\frac{7}{3}\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2-\frac{1}{5}\left[\frac{r_{E/V}}{Z_{E/V}}\right]^2\right)\frac{Z_{E/V}}{r_{E/V}} + \frac{3}{7}D\left(\frac{R_E}{r_{E/V}}\right)^4\left(5-\frac{70}{3}\left[\frac{Z_{E/V}}{r_{E/V}}\right]^2+21\left[\frac{Z_{E/V}}{r_{E/V}}\right]^4\right)\right]$$

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IF:	THEN : SET
$Z_{E/V} = 0$	$\ddot Z'_{E/V} = 0$

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$$P_{X_{E/V}} = \ddot X'_{E/V} + A_{X/V}$$
$$P_{Y_{E/V}} = \ddot Y'_{E/V} + A_{Y/V}$$
$$P_{Z_{E/V}} = \ddot Z'_{E/V} + A_{Z/V}$$

LEM-CSM RELATIVE POSITION

CSM COORDINATES WRT LEM MEASURED IN E OR M-FRAME 10

$$\rho_{x_n} = X_{n/c} - X_{n/L}$$
$$\rho_{Y_n} = Y_{n/c} - Y_{n/L}$$
$$\rho_{Z_n} = Z_{n/c} - Z_{n/L}$$

CSM INERTIAL E OR M-FRAME VELOCITIES WRT LEM 11

$$\dot\rho_{x_n} = \dot X_{n/c} - \dot X_{n/L}$$
$$\dot\rho_{Y_n} = \dot Y_{n/c} - \dot Y_{n/L}$$
$$\dot\rho_{Z_n} = \dot Z_{n/c} - \dot Z_{n/L}$$

AERO ACCELERATIONS

(FOR EARTH MISSIONS ONLY)

VELOCITY OF LEM OR CSM RELATIVE TO ATMOSPHERE 32

$$\dot X_{R/V} = \dot X_{E/V} + \frac{2\pi}{360}\omega_E Y_{E/V}$$
$$\dot Y_{R/V} = \dot Y_{E/V} - \frac{2\pi}{360}\omega_E X_{E/V}$$
$$\dot Z_{R/V} = \dot Z_{E/V}$$
$$V_{R/V} = \left[\dot X_{R/V}^2 + \dot Y_{R/V}^2 + \dot Z_{R/V}^2\right]^{\frac{1}{2}}$$

AERODYNAMIC DRAG PERTURBATIONS 30

$$A_{X/V} = A_V\dot X_{R/V}$$
$$A_{Y/V} = A_V\dot Y_{R/V}$$
$$A_{Z/V} = A_V\dot Z_{R/V}$$

31

$$A_L = -\frac{\rho_L V_{R/L} C_{D_L} S_L}{2m_L}$$
$$A_c = -\frac{\rho_c V_{R/c} C_{D_c} S_c}{2m_c}$$

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ALTITUDE-DENSITY VARIATION LOGIC	
IF:	THEN:
$h_0 \le h_V < h_1$	$\rho_V = \rho_0 e^{-\beta_0 h_V}$
$h_1 \le h_V \le h_2$	$\rho_V = \rho_1 e^{-\beta_1 h_V}$
$h_2 < h_V \le h_3$	$\rho_V = \rho_2 e^{-\beta_2 h_V}$

ALTITUDE ABOVE SPHEROIDAL SURFACE 33

$$h_V = r_{E/V} - R_E\left[1 - f''\sin^2\phi_V\right]$$
$$\sin\phi_V = \frac{Z_{E/V}}{r_{E/V}}$$

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DATE

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C- ROTATIONAL EQUATIONS OF MOTION C

SHEET 2 OF 2

CONSTANTS & INITIAL CONDITIONS 01			
p_{B_0} DEG/SEC	θ_0 DEG	l_1 = 5.0 FT	ϕ_{CSM}
q_{B_0} DEG/SEC	ϕ_0 DEG	l_2 = 5.5 FT	
r_{B_0} DEG/SEC	ψ_0 DEG $\left(-\frac{\pi}{2} \leq \psi_0 \leq \frac{\pi}{2}\right)$		
R = 57.2957795 DEG/RAD		ε =	

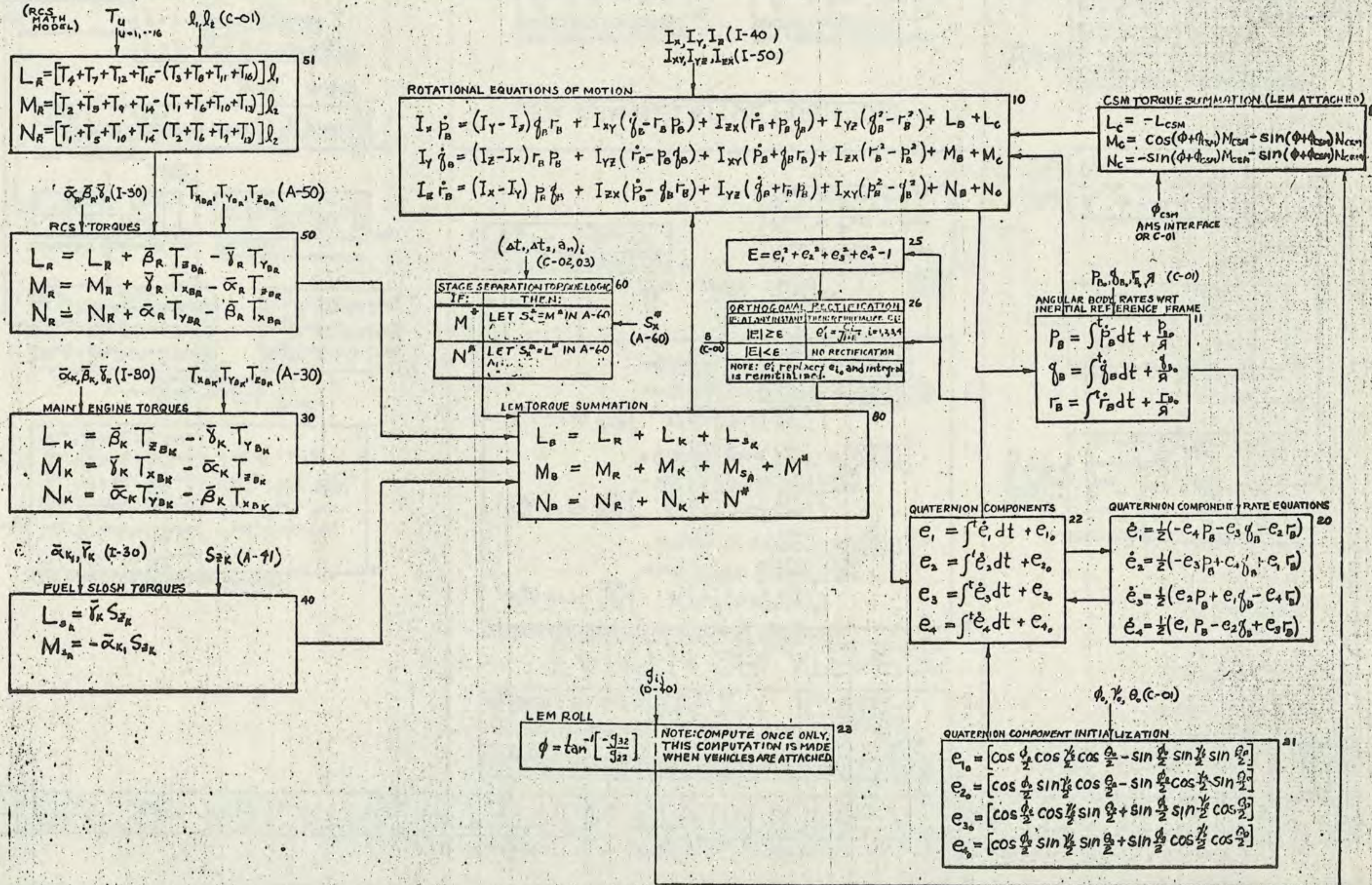
CONSTANTS AND INITIAL CONDITIONS 02

M^*	SUBSCRIPT i PP	FP	LO
t_{1i}			
Δt_{1i}			
Δt_{2i}			
a_{0i}			
a_{1i}			
a_{2i}			
a_{3i}			
a_{4i}			

CONSTANTS AND INITIAL CONDITIONS 03

N^*	SUBSCRIPT i PP	FP	LO
t_{1i}			
Δt_{1i}			
Δt_{2i}			
a_{0i}			
a_{1i}			
a_{2i}			
a_{3i}			
a_{4i}			

SHEET 1 OF 1



$L_R = [T_4+T_7+T_{12}+T_{15}-(T_3+T_8+T_{11}+T_{16})]\,l_1$

$M_R = [T_2+T_5+T_9+T_{14}-(T_1+T_6+T_{10}+T_{13})]\,l_2$

$N_R = [T_1+T_5+T_{10}+T_{14}-(T_2+T_6+T_9+T_{13})]\,l_2$

RCS TORQUES

$L_R = L_R + \bar{\beta}_R T_{z_{B_R}} - \bar{\gamma}_R T_{y_{B_R}}$

$M_R = M_R + \bar{\gamma}_R T_{x_{B_R}} - \bar{\alpha}_R T_{z_{B_R}}$

$N_R = N_R + \bar{\alpha}_R T_{y_{B_R}} - \bar{\beta}_R T_{x_{B_R}}$

MAIN ENGINE TORQUES

$L_K = \bar{\beta}_K T_{z_{B_K}} - \bar{\gamma}_K T_{y_{B_K}}$

$M_K = \bar{\gamma}_K T_{x_{B_K}} - \bar{\alpha}_K T_{z_{B_K}}$

$N_K = \bar{\alpha}_K T_{y_{B_K}} - \bar{\beta}_K T_{x_{B_K}}$

FUEL SLOSH TORQUES

$L_{s_K} = \bar{\gamma}_K S_{z_K}$

$M_{s_K} = -\bar{\alpha}_K S_{z_K}$

ROTATIONAL EQUATIONS OF MOTION

$I_x \dot{p}_B = (I_Y - I_Z) q_B r_B + I_{XY}(\dot{q}_B - r_B p_B) + I_{ZX}(\dot{r}_B + p_B q_B) + I_{YZ}(q_B^2 - r_B^2) + L_B + L_C$

$I_Y \dot{q}_B = (I_Z - I_X) r_B p_B + I_{YZ}(\dot{r}_B - p_B q_B) + I_{XY}(\dot{p}_B + q_B r_B) + I_{ZX}(r_B^2 - p_B^2) + M_B + M_C$

$I_Z \dot{r}_B = (I_X - I_Y) p_B q_B + I_{ZX}(\dot{p}_B - q_B r_B) + I_{YZ}(\dot{q}_B + r_B p_B) + I_{XY}(p_B^2 - q_B^2) + N_B + N_C$

CSM TORQUE SUMMATION (LEM ATTACHED)

$L_C = -L_{CSM}$

$M_C = \cos(\phi+\phi_{CSM})M_{CSM} - \sin(\phi+\phi_{CSM})N_{CSM}$

$N_C = -\sin(\phi+\phi_{CSM})M_{CSM} - \sin(\phi+\phi_{CSM})N_{CSM}$

STAGE SEPARATION TOPSIDE LOGIC

IF:	THEN:
M^*	LET $S_x^* = M^*$ IN A-60
N^*	LET $S_x^* = L^*$ IN A-60

$E = e_1^2 + e_2^2 + e_3^2 + e_4^2 - 1$

ORTHOGONAL RECTIFICATION

IF AT ANY INSTANT	THEN RENORMALIZE e_i
$\lvert E \rvert \geq \varepsilon$	$e_i' = \frac{e_i}{\sqrt{1+E}}$, i=1,2,3,4
$\lvert E \rvert < \varepsilon$	NO RECTIFICATION

NOTE: e_i' replaces e_{i_0} and integral is reinitialized.

LEM TORQUE SUMMATION

$L_B = L_R + L_K + L_{s_K}$

$M_B = M_R + M_K + M_{s_K} + M^*$

$N_B = N_R + N_K + N^*$

ANGULAR BODY RATES WRT INERTIAL REFERENCE FRAME

$p_B = \int^t \dot{p}_B\,dt + \frac{p_{B_0}}{R}$

$q_B = \int^t \dot{q}_B\,dt + \frac{q_{B_0}}{R}$

$r_B = \int^t \dot{r}_B\,dt + \frac{r_{B_0}}{R}$

QUATERNION COMPONENTS

$e_1 = \int^t \dot{e}_1\,dt + e_{1_0}$

$e_2 = \int^t \dot{e}_2\,dt + e_{2_0}$

$e_3 = \int^t \dot{e}_3\,dt + e_{3_0}$

$e_4 = \int^t \dot{e}_4\,dt + e_{4_0}$

QUATERNION COMPONENT RATE EQUATIONS

$\dot{e}_1 = \frac{1}{2}(-e_4 p_B - e_3 q_B - e_2 r_B)$

$\dot{e}_2 = \frac{1}{2}(-e_3 p_B + e_4 q_B + e_1 r_B)$

$\dot{e}_3 = \frac{1}{2}(e_2 p_B + e_1 q_B - e_4 r_B)$

$\dot{e}_4 = \frac{1}{2}(e_1 p_B - e_2 q_B + e_3 r_B)$

LEM ROLL

$\phi = \tan^{-1}\left[\frac{-g_{32}}{g_{22}}\right]$

NOTE: COMPUTE ONCE ONLY. THIS COMPUTATION IS MADE WHEN VEHICLES ARE ATTACHED.

QUATERNION COMPONENT INITIALIZATION

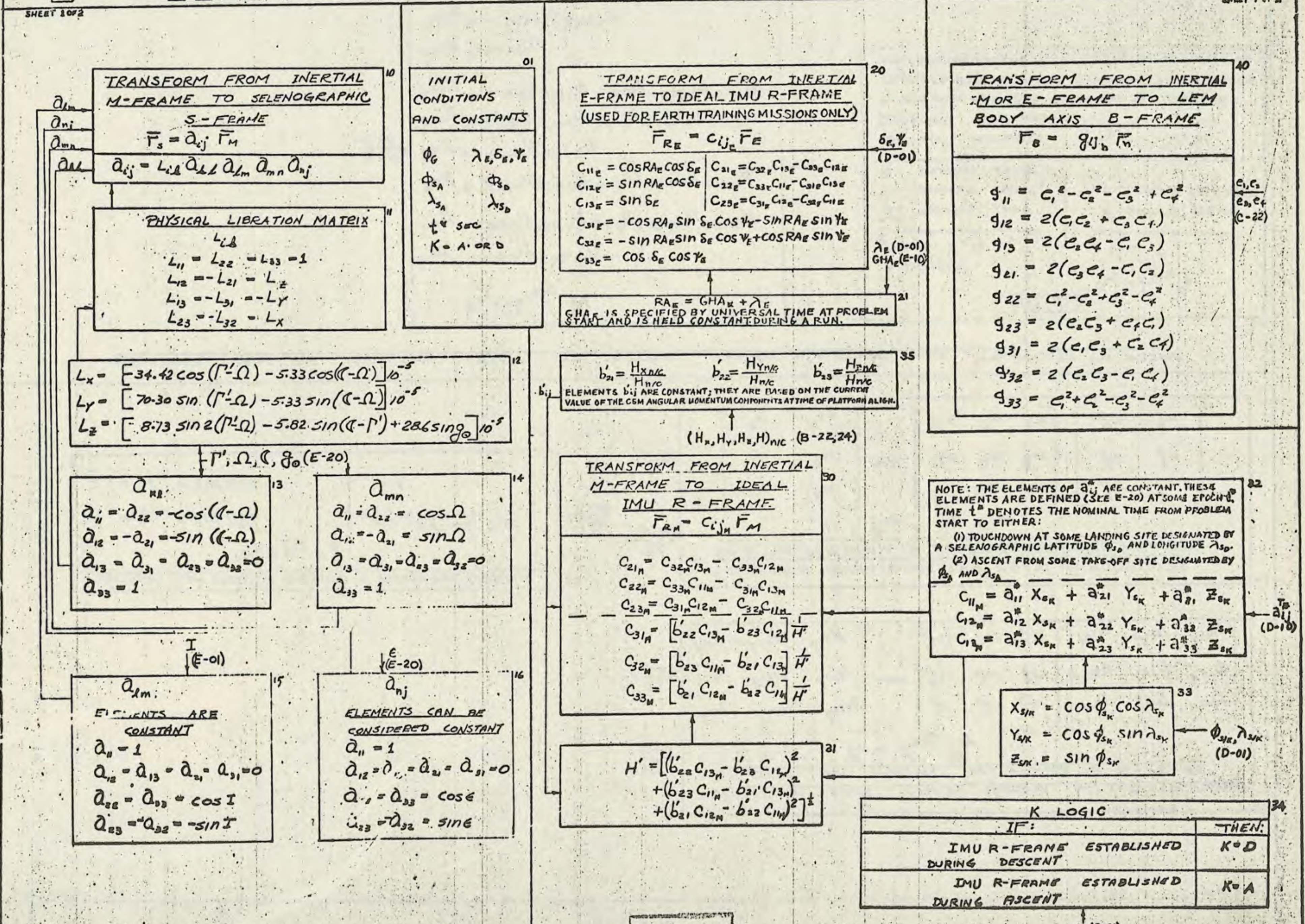
$e_{1_0} = \left[\cos\frac{\phi_0}{2}\cos\frac{\psi_0}{2}\cos\frac{\theta_0}{2} - \sin\frac{\phi_0}{2}\sin\frac{\psi_0}{2}\sin\frac{\theta_0}{2}\right]$

$e_{2_0} = \left[\cos\frac{\phi_0}{2}\sin\frac{\psi_0}{2}\cos\frac{\theta_0}{2} - \sin\frac{\phi_0}{2}\cos\frac{\psi_0}{2}\sin\frac{\theta_0}{2}\right]$

$e_{3_0} = \left[\cos\frac{\phi_0}{2}\cos\frac{\psi_0}{2}\sin\frac{\theta_0}{2} + \sin\frac{\phi_0}{2}\sin\frac{\psi_0}{2}\cos\frac{\theta_0}{2}\right]$

$e_{4_0} = \left[\cos\frac{\phi_0}{2}\sin\frac{\psi_0}{2}\sin\frac{\theta_0}{2} + \sin\frac{\phi_0}{2}\cos\frac{\psi_0}{2}\cos\frac{\theta_0}{2}\right]$

D- GENERAL TRANSFORMATIONS



10

TRANSFORM FROM INERTIAL M-FRAME TO SELENOGRAPHIC S-FRAME

$$\overline{r}_s = a_{ij}\,\overline{r}_M$$

$$a_{ij} = L_{i\ell}\, a_{\ell\ell}\, a_{\ell m}\, a_{mn}\, a_{nj}$$

01

INITIAL CONDITIONS AND CONSTANTS

ϕ_G $\lambda_E, \delta_E, \psi_E$
ϕ_{s_A} ϕ_{s_D}
λ_{s_A} λ_{s_D}
t^{*} sec
K = A OR D

20

TRANSFORM FROM INERTIAL E-FRAME TO IDEAL IMU R-FRAME (USED FOR EARTH TRAINING MISSIONS ONLY)

$$\overline{r}_{R_E} = c_{ij_E}\,\overline{r}_E$$

$C_{11_E} = \cos RA_E \cos \delta_E$ $C_{21_E} = C_{32_E}C_{13_E} - C_{33_E}C_{12_E}$
$C_{12_E} = \sin RA_E \cos \delta_E$ $C_{22_E} = C_{33_E}C_{11_E} - C_{31_E}C_{13_E}$
$C_{13_E} = \sin \delta_E$ $C_{23_E} = C_{31_E}C_{12_E} - C_{32_E}C_{11_E}$
$C_{31_E} = -\cos RA_E \sin \delta_E \cos \psi_E - \sin RA_E \sin \psi_E$
$C_{32_E} = -\sin RA_E \sin \delta_E \cos \psi_E + \cos RA_E \sin \psi_E$
$C_{33_E} = \cos \delta_E \cos \psi_E$

21

$$RA_E = GHA_E + \lambda_E$$

GHA

GHA_E IS SPECIFIED BY UNIVERSAL TIME AT PROBLEM START AND IS HELD CONSTANT DURING A RUN.

40

TRANSFORM FROM INERTIAL M OR E-FRAME TO LEM BODY AXIS B-FRAME

$$\overline{r}_B = g_{ij_b}\,\overline{r}_n$$

$g_{11} = e_1^2 - e_2^2 - e_3^2 + e_4^2$
$g_{12} = 2(e_1e_2 + e_3e_4)$
$g_{13} = 2(e_2e_4 - e_1e_3)$
$g_{21} = 2(e_3e_4 - e_1e_2)$
$g_{22} = e_1^2 - e_2^2 + e_3^2 - e_4^2$
$g_{23} = 2(e_2e_3 + e_4e_1)$
$g_{31} = 2(e_1e_3 + e_2e_4)$
$g_{32} = 2(e_2e_3 - e_1e_4)$
$g_{33} = e_1^2 + e_2^2 - e_3^2 - e_4^2$

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PHYSICAL LIBRATION MATRIX

$L_{i\ell}$
$L_{11} = L_{22} = L_{33} = 1$
$L_{12} = -L_{21} = L_Z$
$L_{13} = -L_{31} = -L_Y$
$L_{23} = -L_{32} = L_X$

12

$L_X = [34.42\cos(\Gamma' - \Omega) - 5.33\cos(\leftmoon - \Omega)]10^{-5}$
$L_Y = [70.30\sin(\Gamma' - \Omega) - 5.33\sin(\leftmoon - \Omega)]10^{-5}$
$L_Z = [8.73\sin 2(\Gamma' - \Omega) - 5.82\sin(\leftmoon - \Gamma') + 2.86\sin g_0]10^{-5}$

35

$b'_{21} = \dfrac{H_{X_{n/c}}}{H_{n/c}}$ $b'_{22} = \dfrac{H_{Y_{n/c}}}{H_{n/c}}$ $b'_{23} = \dfrac{H_{Z_{n/c}}}{H_{n/c}}$

ELEMENTS b'_{ij} ARE CONSTANT; THEY ARE BASED ON THE CURRENT VALUE OF THE CSM ANGULAR MOMENTUM COMPONENTS AT TIME OF PLATFORM ALIGN.

13

$a_{\ell\ell}$
$a_{11} = a_{22} = -\cos(\leftmoon - \Omega)$
$a_{12} = -a_{21} = -\sin(\leftmoon - \Omega)$
$a_{13} = a_{31} = a_{23} = a_{32} = 0$
$a_{33} = 1$

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a_{mn}
$a_{11} = a_{22} = \cos\Omega$
$a_{12} = -a_{21} = \sin\Omega$
$a_{13} = a_{31} = a_{23} = a_{32} = 0$
$a_{33} = 1$

30

TRANSFORM FROM INERTIAL M-FRAME TO IDEAL IMU R-FRAME

$$\overline{r}_{R_M} = C_{ij_M}\,\overline{r}_M$$

$C_{21_M} = C_{32_M}C_{13_M} - C_{33_M}C_{12_M}$
$C_{22_M} = C_{33_M}C_{11_M} - C_{31_M}C_{13_M}$
$C_{23_M} = C_{31_M}C_{12_M} - C_{32_M}C_{11_M}$
$C_{31_M} = [b'_{22}C_{13_M} - b'_{23}C_{12_M}]\dfrac{1}{H'}$
$C_{32_M} = [b'_{23}C_{11_M} - b'_{21}C_{13_M}]\dfrac{1}{H'}$
$C_{33_M} = [b'_{21}C_{12_M} - b'_{22}C_{11_M}]\dfrac{1}{H'}$

32

NOTE: THE ELEMENTS OF a^{*}_{ij} ARE CONSTANT. THESE ELEMENTS ARE DEFINED (SEE E-20) AT SOME EPOCH t^{*}. TIME t^{*} DENOTES THE NOMINAL TIME FROM PROBLEM START TO EITHER:

(1) TOUCHDOWN AT SOME LANDING SITE DESIGNATED BY A SELENOGRAPHIC LATITUDE ϕ_{s_D} AND LONGITUDE λ_{s_D}.

(2) ASCENT FROM SOME TAKE-OFF SITE DESIGNATED BY ϕ_{s_A} AND λ_{s_A}

$C_{11_M} = a^{*}_{11}X_{s_K} + a^{*}_{21}Y_{s_K} + a^{*}_{31}Z_{s_K}$
$C_{12_M} = a^{*}_{12}X_{s_K} + a^{*}_{22}Y_{s_K} + a^{*}_{32}Z_{s_K}$
$C_{13_M} = a^{*}_{13}X_{s_K} + a^{*}_{23}Y_{s_K} + a^{*}_{33}Z_{s_K}$

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$a_{\ell m}$:

ELEMENTS ARE CONSTANT

$a_{11} = 1$
$a_{12} = a_{13} = a_{21} = a_{31} = 0$
$a_{22} = a_{33} = \cos I$
$a_{23} = -a_{32} = -\sin I$

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a_{nj}

ELEMENTS CAN BE CONSIDERED CONSTANT

$a_{11} = 1$
$a_{12} = a_{13} = a_{21} = a_{31} = 0$
$a_{22} = a_{33} = \cos\epsilon$
$a_{23} = -a_{32} = \sin\epsilon$

31

$$H' = \left[(b'_{22}C_{13_M} - b'_{23}C_{12_M})^2 + (b'_{23}C_{11_M} - b'_{21}C_{13_M})^2 + (b'_{21}C_{12_M} - b'_{22}C_{11_M})^2\right]^{\frac{1}{2}}$$

33

$X_{s_K} = \cos\phi_{s_K}\cos\lambda_{s_K}$
$Y_{s_K} = \cos\phi_{s_K}\sin\lambda_{s_K}$
$Z_{s_K} = \sin\phi_{s_K}$

34

K LOGIC	
IF:	THEN:
IMU R-FRAME ESTABLISHED DURING DESCENT	K = D
IMU R-FRAME ESTABLISHED DURING ASCENT	K = A

1664

80

$h_{ij_{pg}}$ (D-70)

g_{ij_n} (D-40)

TRANSFORM FROM OPTICAL WINDOW W- OR TELESCOPE T- FRAME TO INERTIAL M- FRAME OR E-FRAME

$$\bar{r}_n = l_{ij_{pg_n}} \bar{r}_{pg}$$

$$\begin{bmatrix} l_{11pg} & l_{12pg} & l_{13pg} \\ l_{21pg} & l_{22pg} & l_{23pg} \\ l_{31pg} & l_{32pg} & l_{33pg} \end{bmatrix}_n = \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ g_{12} & g_{22} & g_{32} \\ g_{13} & g_{23} & g_{33} \end{bmatrix}_n \begin{bmatrix} h_{11pg} & h_{21pg} & h_{31pg} \\ h_{12pg} & h_{22pg} & h_{32pg} \\ h_{13pg} & h_{23pg} & h_{33pg} \end{bmatrix}$$

60

TRANSFORM FROM INERTIAL E-FRAME TO GEOGRAPHIC G-FRAME

$$\bar{r}_G = f_{ij} \bar{r}_E$$

$f_{11} = f_{22} = \cos GHA$

$f_{12} = -f_{21} = \sin GHA$

$f_{13} = f_{23} = f_{31} = f_{32} = 0$

$f_{33} = 1$

GHA (E-10)

90

g_{ij_M} (D-40)

a_{ij} (D-10)

TRANSFORM FROM ALTIMETER BEAM IN BODY AXES TO ALTIMETER BEAM IN SELENOGRAPHIC COORDINATES

$$\bar{r}_{S/A} = s_{ij} \bar{r}_{B/A}$$

$$\begin{bmatrix} s_{11} & s_{13} \\ s_{21} & s_{23} \\ s_{31} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} g_{11} & g_{31} \\ g_{12} & g_{32} \\ g_{13} & g_{33} \end{bmatrix}_M$$

TRANSFORM FROM THE LEM BODY B-FRAME TO OPTICAL WINDOW W-FRAME OR TELESCOPE T-FRAME

70

$(\phi,\theta,\psi)_{pg}$ (J-01)

$$\bar{r}_{pg} = h_{ij_{pg}} \bar{r}_B$$

$h_{11pg} = \cos\psi_{pg} \cos\theta_{pg}$

$h_{12pg} = \cos\psi_{pg} \sin\theta_{pg} \sin\phi_{pg} + \sin\psi_{pg} \cos\phi_{pg}$

$h_{13pg} = -\cos\psi_{pg} \sin\theta_{pg} \cos\phi_{pg} + \sin\psi_{pg} \sin\phi_{pg}$

$h_{21pg} = -\sin\psi_{pg} \cos\theta_{pg}$

$h_{22pg} = -\sin\psi_{pg} \sin\theta_{pg} \sin\phi_{pg} + \cos\psi_{pg} \cos\phi_{pg}$

$h_{23pg} = \sin\psi_{pg} \sin\theta_{pg} \cos\phi_{pg} + \cos\psi_{pg} \sin\phi_{pg}$

$h_{31pg} = \sin\theta_{pg}$

$h_{32pg} = -\cos\theta_{pg} \sin\phi_{pg}$

$h_{33pg} = \cos\theta_{pg} \cos\phi_{pg}$

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NOTES

$$\bar{r}_{pg} = h_{ij_{pg}} \bar{r}_B$$

SUBSCRIPTS:

$g = l, r, a$

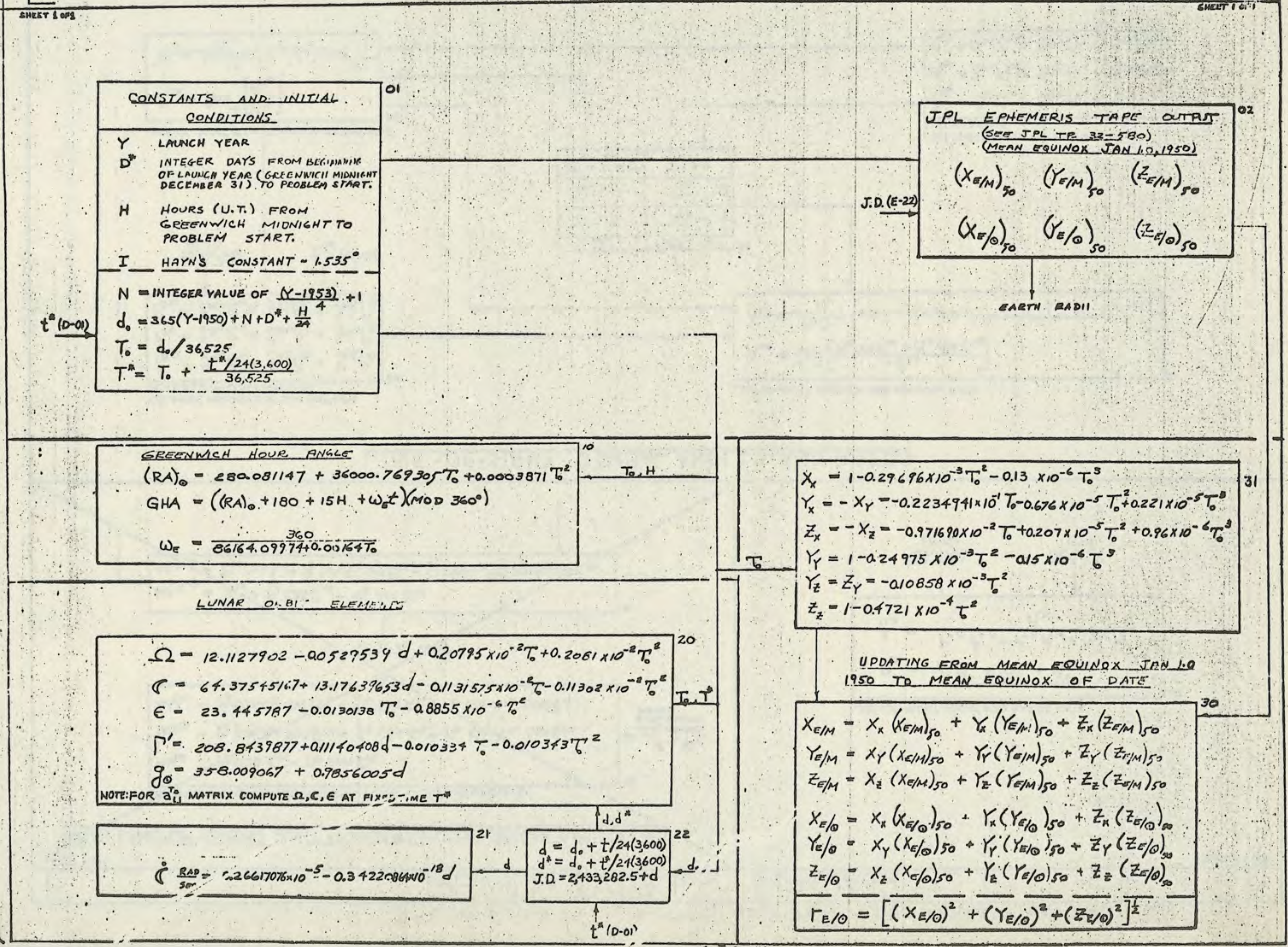
l DENOTES LEFT
r DENOTES RIGHT
a DENOTES ABOVE

$p = W, T$

W DENOTES WINDOW
T DENOTES TELESCOPE

WINDOW MATRICES l, r, a WILL BE COMPUTED CONCURRENTLY. AT ANY ONE TIME EITHER THE $l, r,$ OR a TELESCOPE MATRIX WILL BE COMPUTED. ANGULAR INPUTS PERTAINING TO THE WINDOW AND TELESCOPE OPTICAL AXES $(\phi,\theta,\psi)_{pg}$ ARE GIVEN IN J-01.

E- EPHEMERIS E



CONSTANTS AND INITIAL CONDITIONS

Y LAUNCH YEAR

D^* INTEGER DAYS FROM BEGINNING OF LAUNCH YEAR (GREENWICH MIDNIGHT DECEMBER 31) TO PROBLEM START.

H HOURS (U.T.) FROM GREENWICH MIDNIGHT TO PROBLEM START.

I HAYN'S CONSTANT - 1.535°

N = INTEGER VALUE OF $\frac{(Y-1953)}{4}+1$

$$d_o = 365(Y-1950)+N+D^*+\frac{H}{24}$$

$$T_o = d_o/36{,}525$$

$$T^* = T_o + \frac{t^*/24(3{,}600)}{36{,}525}$$

JPL EPHEMERIS TAPE OUTPUT
(SEE JPL TR 32-580)
(MEAN EQUINOX JAN 1.0, 1950)

$$(X_{E/M})_{50} \quad (Y_{E/M})_{50} \quad (Z_{E/M})_{50}$$

$$(X_{E/\odot})_{50} \quad (Y_{E/\odot})_{50} \quad (Z_{E/\odot})_{50}$$

GREENWICH HOUR ANGLE

$$(RA)_\odot = 280.081147 + 36000.769305\,T_o + 0.0003871\,T_o^2$$

$$GHA = \left((RA)_\odot + 180 + 15H + \omega_E t\right)(\text{MOD } 360°)$$

$$\omega_E = \frac{360}{86164.09974 + 0.00164\,T_o}$$

$$X_x = 1 - 0.29696\times10^{-3}T_o^2 - 0.13\times10^{-6}T_o^3$$

$$Y_x = -X_Y = -0.2234941\times10^{-1}T_o - 0.676\times10^{-5}T_o^2 + 0.221\times10^{-5}T_o^3$$

$$Z_x = -X_z = -0.971690\times10^{-2}T_o + 0.207\times10^{-5}T_o^2 + 0.96\times10^{-6}T_o^3$$

$$Y_Y = 1 - 0.24975\times10^{-3}T_o^2 - 0.15\times10^{-6}T_o^3$$

$$Y_z = Z_Y = -0.10858\times10^{-3}T_o^2$$

$$Z_z = 1 - 0.4721\times10^{-4}T_o^2$$

LUNAR ORBIT ELEMENTS

$$\Omega = 12.1127902 - 0.05295394\,d + 0.20795\times10^{-2}T_o + 0.2061\times10^{-2}T_o^2$$

$$☾ = 64.37545167 + 13.17639653\,d - 0.1131575\times10^{-2}T_o - 0.11302\times10^{-2}T_o^2$$

$$\epsilon = 23.445787 - 0.0130138\,T_o - 0.8855\times10^{-6}T_o^2$$

$$\Gamma' = 208.8439877 + 0.11140408\,d - 0.010334\,T_o - 0.010343\,T_o^2$$

$$g_\odot = 358.009067 + 0.9856005\,d$$

NOTE: FOR $a_{ij}^{T_o}$ MATRIX COMPUTE Ω, ☾, ε AT FIXED TIME T^*

$$\dot{☾}\ \frac{\text{RAD}}{\text{SEC}} = 0.26617076\times10^{-5} - 0.34220864\times10^{-18}\,d$$

$$d = d_o + t/24(3{,}600)$$

$$d^* = d_o + t^*/24(3{,}600)$$

$$J.D. = 2{,}433{,}282.5 + d$$

UPDATING FROM MEAN EQUINOX JAN 1.0 1950 TO MEAN EQUINOX OF DATE

$$X_{E/M} = X_x(X_{E/M})_{50} + Y_x(Y_{E/M})_{50} + Z_x(Z_{E/M})_{50}$$

$$Y_{E/M} = X_Y(X_{E/M})_{50} + Y_Y(Y_{E/M})_{50} + Z_Y(Z_{E/M})_{50}$$

$$Z_{E/M} = X_z(X_{E/M})_{50} + Y_z(Y_{E/M})_{50} + Z_z(Z_{E/M})_{50}$$

$$X_{E/\odot} = X_x(X_{E/\odot})_{50} + Y_x(Y_{E/\odot})_{50} + Z_x(Z_{E/\odot})_{50}$$

$$Y_{E/\odot} = X_Y(X_{E/\odot})_{50} + Y_Y(Y_{E/\odot})_{50} + Z_Y(Z_{E/\odot})_{50}$$

$$Z_{E/\odot} = X_z(X_{E/\odot})_{50} + Y_z(Y_{E/\odot})_{50} + Z_z(Z_{E/\odot})_{50}$$

$$r_{E/\odot} = \left[(X_{E/\odot})^2 + (Y_{E/\odot})^2 + (Z_{E/\odot})^2\right]^{\frac{1}{2}}$$

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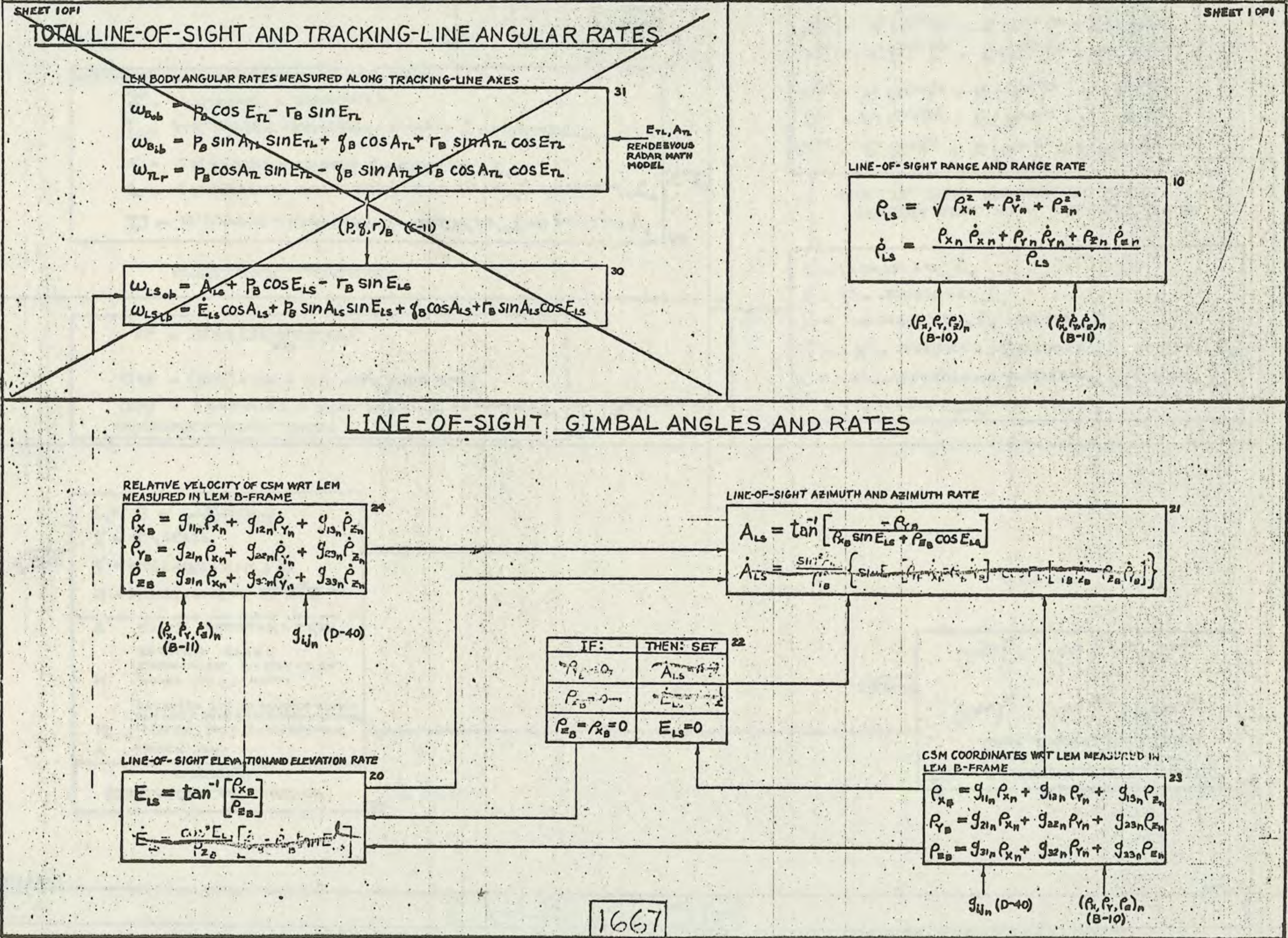
F

RENDEZVOUS RADAR

F



SHEET 1 OF 1

~~TOTAL LINE-OF-SIGHT AND TRACKING-LINE ANGULAR RATES~~

~~LEM BODY ANGULAR RATES MEASURED ALONG TRACKING-LINE AXES~~

31

~~$\omega_{B_{ob}} = p_B \cos E_{TL} - r_B \sin E_{TL}$~~

~~$\omega_{B_{ib}} = p_B \sin A_{TL} \sin E_{TL} + q_B \cos A_{TL} + r_B \sin A_{TL} \cos E_{TL}$~~

~~$\omega_{TL_r} = p_B \cos A_{TL} \sin E_{TL} - q_B \sin A_{TL} + r_B \cos A_{TL} \cos E_{TL}$~~

30

~~$\omega_{LS_{ob}} = \dot{A}_{LS} + p_B \cos E_{LS} - r_B \sin E_{LS}$~~

~~$\omega_{LS_{ib}} = \dot{E}_{LS} \cos A_{LS} + p_B \sin A_{LS} \sin E_{LS} + q_B \cos A_{LS} + r_B \sin A_{LS} \cos E_{LS}$~~

SHEET 1 OF 1

LINE-OF-SIGHT RANGE AND RANGE RATE

10

$$\rho_{LS} = \sqrt{\rho_{X_n}^2 + \rho_{Y_n}^2 + \rho_{Z_n}^2}$$

$$\dot{\rho}_{LS} = \frac{\rho_{X_n}\dot{\rho}_{X_n} + \rho_{Y_n}\dot{\rho}_{Y_n} + \rho_{Z_n}\dot{\rho}_{Z_n}}{\rho_{LS}}$$

LINE-OF-SIGHT GIMBAL ANGLES AND RATES

RELATIVE VELOCITY OF CSM WRT LEM MEASURED IN LEM B-FRAME

24

$$\dot{\rho}_{X_B} = g_{11_n}\dot{\rho}_{X_n} + g_{12_n}\dot{\rho}_{Y_n} + g_{13_n}\dot{\rho}_{Z_n}$$

$$\dot{\rho}_{Y_B} = g_{21_n}\dot{\rho}_{X_n} + g_{22_n}\dot{\rho}_{Y_n} + g_{23_n}\dot{\rho}_{Z_n}$$

$$\dot{\rho}_{Z_B} = g_{31_n}\dot{\rho}_{X_n} + g_{32_n}\dot{\rho}_{Y_n} + g_{33_n}\dot{\rho}_{Z_n}$$

LINE-OF-SIGHT AZIMUTH AND AZIMUTH RATE

21

$$A_{LS} = \tan^{-1}\left[\frac{-\rho_{Y_B}}{\rho_{X_B} \sin E_{LS} + \rho_{Z_B} \cos E_{LS}}\right]$$

~~$\dot{A}_{LS} = \frac{\sin^2 [illegible]}{[illegible]}\{ [illegible] \}$~~

22

IF:	THEN: SET
~~[illegible]~~	~~$\dot{A}_{LS}$ [illegible]~~
~~[illegible]~~	~~$\dot{E}_{LS}$ [illegible]~~
$\rho_{Z_B} = \rho_{X_B} = 0$	$E_{LS} = 0$

LINE-OF-SIGHT ELEVATION AND ELEVATION RATE

20

$$E_{LS} = \tan^{-1}\left[\frac{\rho_{X_B}}{\rho_{Z_B}}\right]$$

~~$\dot{E}_{LS} = \frac{\cos^2 E_{LS}}{\rho_{Z_B}}\left[[illegible] \tan E_{LS} \right]$~~

CSM COORDINATES WRT LEM MEASURED IN LEM B-FRAME

23

$$\rho_{X_B} = g_{11_n}\rho_{X_n} + g_{12_n}\rho_{Y_n} + g_{13_n}\rho_{Z_n}$$

$$\rho_{Y_B} = g_{21_n}\rho_{X_n} + g_{22_n}\rho_{Y_n} + g_{23_n}\rho_{Z_n}$$

$$\rho_{Z_B} = g_{31_n}\rho_{X_n} + g_{32_n}\rho_{Y_n} + g_{33_n}\rho_{Z_n}$$

G-

SHEET 1 OF 2

LEM LUNAR LANDING RADAR

G

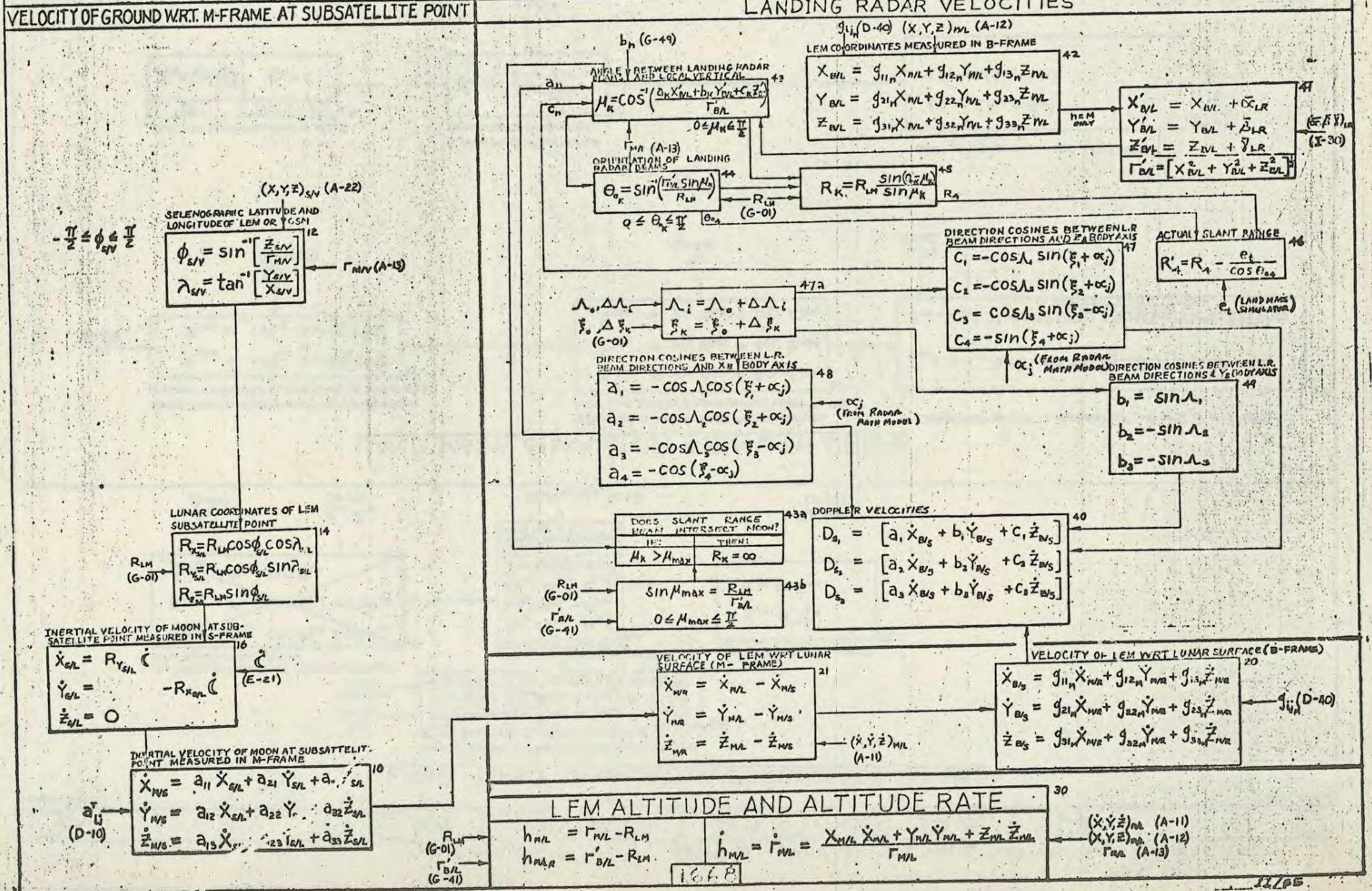
SHEET 1 OF 2

CONSTANTS AND INITIAL CONDITIONS

01

θ_{LM}, S_{LM} $K_0 = 0.9952557 \times 10^5$ ft/deg	$\Lambda_0 = 13.0°$ $\xi_0 = 20.6°$	$\phi_{LM_\ell}, \lambda_{LM_\ell}$ R_{LM_ℓ} $\ell = 1, 2, \ldots, 10$	$\Delta\xi_1$ $\Delta\xi_2$ $\Delta\xi_3$ $\Delta\xi_4$	$\Delta\Lambda_1$ $\Delta\Lambda_2$ $\Delta\Lambda_3$

SUBSCRIPT	REFERS TO
$i = 1, 2, 3$	DOPPLER VELOCITY BEAM
$j = 1, 2$	ANTENNA TILT ANGLE
$K = 1, 2, 3, 4$	VELOCITY AND ALTITUDE BEAMS



VELOCITY OF GROUND W.R.T. M-FRAME AT SUBSATELLITE POINT

SELENOGRAPHIC LATITUDE AND LONGITUDE OF LEM OR CSM

12

$$\phi_{S/V} = \sin^{-1}\left[\frac{Z_{S/V}}{r_{M/V}}\right]$$

$$\lambda_{S/V} = \tan^{-1}\left[\frac{Y_{S/V}}{X_{S/V}}\right]$$

$$-\frac{\pi}{2} \leq \phi_{S/V} \leq \frac{\pi}{2}$$

LUNAR COORDINATES OF LEM SUBSATELLITE POINT

14

$$R_{X_{S/L}} = R_{LM}\cos\phi_{S/L}\cos\lambda_{S/L}$$

$$R_{Y_{S/L}} = R_{LM}\cos\phi_{S/L}\sin\lambda_{S/L}$$

$$R_{Z_{S/L}} = R_{LM}\sin\phi_{S/L}$$

INERTIAL VELOCITY OF MOON AT SUBSATELLITE POINT MEASURED IN S-FRAME

16

$$\dot{X}_{S/L} = R_{Y_{S/L}}\dot{☾}$$

$$\dot{Y}_{S/L} = -R_{X_{S/L}}\dot{☾}$$

$$\dot{Z}_{S/L} = 0$$

INERTIAL VELOCITY OF MOON AT SUBSATTELITE POINT MEASURED IN M-FRAME

10

$$\dot{X}_{M/S} = a_{11}\dot{X}_{S/L} + a_{21}\dot{Y}_{S/L} + a_{31}\dot{Z}_{S/L}$$

$$\dot{Y}_{M/S} = a_{12}\dot{X}_{S/L} + a_{22}\dot{Y}_{S/L} + a_{32}\dot{Z}_{S/L}$$

$$\dot{Z}_{M/S} = a_{13}\dot{X}_{S/L} + a_{23}\dot{Y}_{S/L} + a_{33}\dot{Z}_{S/L}$$

LANDING RADAR VELOCITIES

LEM COORDINATES MEASURED IN B-FRAME

42

$$X_{B/L} = g_{11_n}X_{M/L} + g_{12_n}Y_{M/L} + g_{13_n}Z_{M/L}$$

$$Y_{B/L} = g_{21_n}X_{M/L} + g_{22_n}Y_{M/L} + g_{23_n}Z_{M/L}$$

$$Z_{B/L} = g_{31_n}X_{M/L} + g_{32_n}Y_{M/L} + g_{33_n}Z_{M/L}$$

41

$$X'_{B/L} = X_{B/L} + \bar{\alpha}_{LR}$$

$$Y'_{B/L} = Y_{B/L} + \bar{\beta}_{LR}$$

$$Z'_{B/L} = Z_{B/L} + \bar{\gamma}_{LR}$$

$$r'_{B/L} = \left[X^2_{B/L} + Y^2_{B/L} + Z^2_{B/L}\right]^{1/2}$$

ANGLE BETWEEN LANDING RADAR BEAMS AND LOCAL VERTICAL

43

$$\mu_K = \cos^{-1}\left(\frac{a_K X'_{B/L} + b_K Y'_{B/L} + c_K Z'_{B/L}}{r'_{B/L}}\right)$$

$$0 \leq \mu_K \leq \frac{\pi}{2}$$

ORIENTATION OF LANDING RADAR BEAMS

44

$$\theta_{o_K} = \sin^{-1}\left(\frac{r'_{B/L}\sin\mu_K}{R_{LM}}\right)$$

$$0 \leq \theta_{o_K} \leq \frac{\pi}{2}$$

45

$$R_K = R_{LM}\frac{\sin(\theta_{o_K} - \mu_K)}{\sin\mu_K}$$

ACTUAL SLANT RANGE

46

$$R'_4 = R_4 - \frac{e_t}{\cos\theta_{o_4}}$$

47a

$$\Lambda_i = \Lambda_0 + \Delta\Lambda_i$$

$$\xi_K = \xi_0 + \Delta\xi_K$$

DIRECTION COSINES BETWEEN L.R. BEAM DIRECTIONS AND Z_B BODY AXIS

47

$$c_1 = -\cos\Lambda_1\sin(\xi_1 + \alpha_j)$$

$$c_2 = -\cos\Lambda_2\sin(\xi_2 + \alpha_j)$$

$$c_3 = \cos\Lambda_3\sin(\xi_3 - \alpha_j)$$

$$c_4 = -\sin(\xi_4 + \alpha_j)$$

DIRECTION COSINES BETWEEN L.R. BEAM DIRECTIONS AND X_B BODY AXIS

48

$$a_1 = -\cos\Lambda_1\cos(\xi_1 + \alpha_j)$$

$$a_2 = -\cos\Lambda_2\cos(\xi_2 + \alpha_j)$$

$$a_3 = -\cos\Lambda_3\cos(\xi_3 - \alpha_j)$$

$$a_4 = -\cos(\xi_4 - \alpha_j)$$

DIRECTION COSINES BETWEEN L.R. BEAM DIRECTIONS & Y_B BODY AXIS

49

$$b_1 = \sin\Lambda_1$$

$$b_2 = -\sin\Lambda_2$$

$$b_3 = -\sin\Lambda_3$$

43a

DOES SLANT RANGE BEAM INTERSECT MOON?	
IF:	THEN:
$\mu_K > \mu_{max}$	$R_K = \infty$

43b

$$\sin\mu_{max} = \frac{R_{LM}}{r'_{B/L}}$$

$$0 \leq \mu_{max} \leq \frac{\pi}{2}$$

DOPPLER VELOCITIES

40

$$D_{S_1} = \left[a_1\dot{X}_{B/S} + b_1\dot{Y}_{B/S} + c_1\dot{Z}_{B/S}\right]$$

$$D_{S_2} = \left[a_2\dot{X}_{B/S} + b_2\dot{Y}_{B/S} + c_2\dot{Z}_{B/S}\right]$$

$$D_{S_3} = \left[a_3\dot{X}_{B/S} + b_3\dot{Y}_{B/S} + c_3\dot{Z}_{B/S}\right]$$

VELOCITY OF LEM WRT LUNAR SURFACE (M-FRAME)

21

$$\dot{X}_{M/R} = \dot{X}_{M/L} - \dot{X}_{M/S}$$

$$\dot{Y}_{M/R} = \dot{Y}_{M/L} - \dot{Y}_{M/S}$$

$$\dot{Z}_{M/R} = \dot{Z}_{M/L} - \dot{Z}_{M/S}$$

VELOCITY OF LEM WRT LUNAR SURFACE (B-FRAME)

20

$$\dot{X}_{B/S} = g_{11_M}\dot{X}_{M/R} + g_{12_M}\dot{Y}_{M/R} + g_{13_M}\dot{Z}_{M/R}$$

$$\dot{Y}_{B/S} = g_{21_M}\dot{X}_{M/R} + g_{22_M}\dot{Y}_{M/R} + g_{23_M}\dot{Z}_{M/R}$$

$$\dot{Z}_{B/S} = g_{31_M}\dot{X}_{M/R} + g_{32_M}\dot{Y}_{M/R} + g_{33_M}\dot{Z}_{M/R}$$

LEM ALTITUDE AND ALTITUDE RATE

30

$$h_{M/L} = r_{M/L} - R_{LM}$$

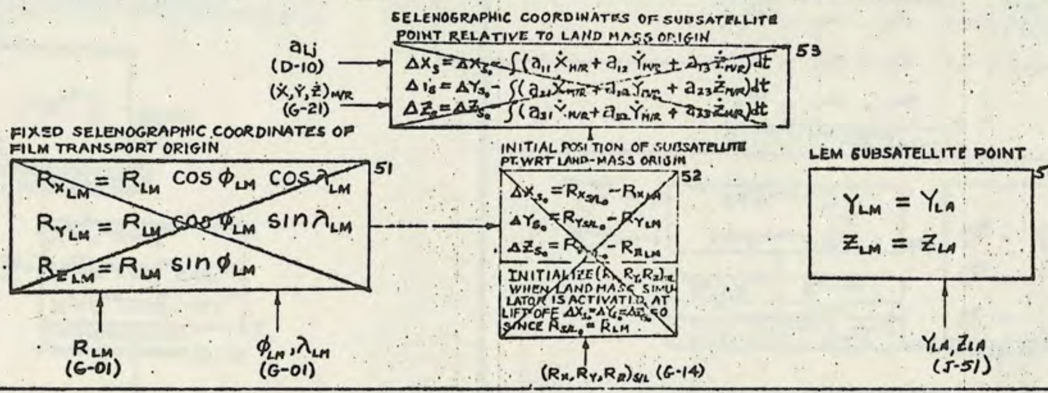
$$h_{M/L_R} = r'_{B/L} - R_{LM}$$

$$\dot{h}_{M/L} = \dot{r}_{M/L} = \frac{X_{M/L}\dot{X}_{M/L} + Y_{M/L}\dot{Y}_{M/L} + Z_{M/L}\dot{Z}_{M/L}}{r_{M/L}}$$

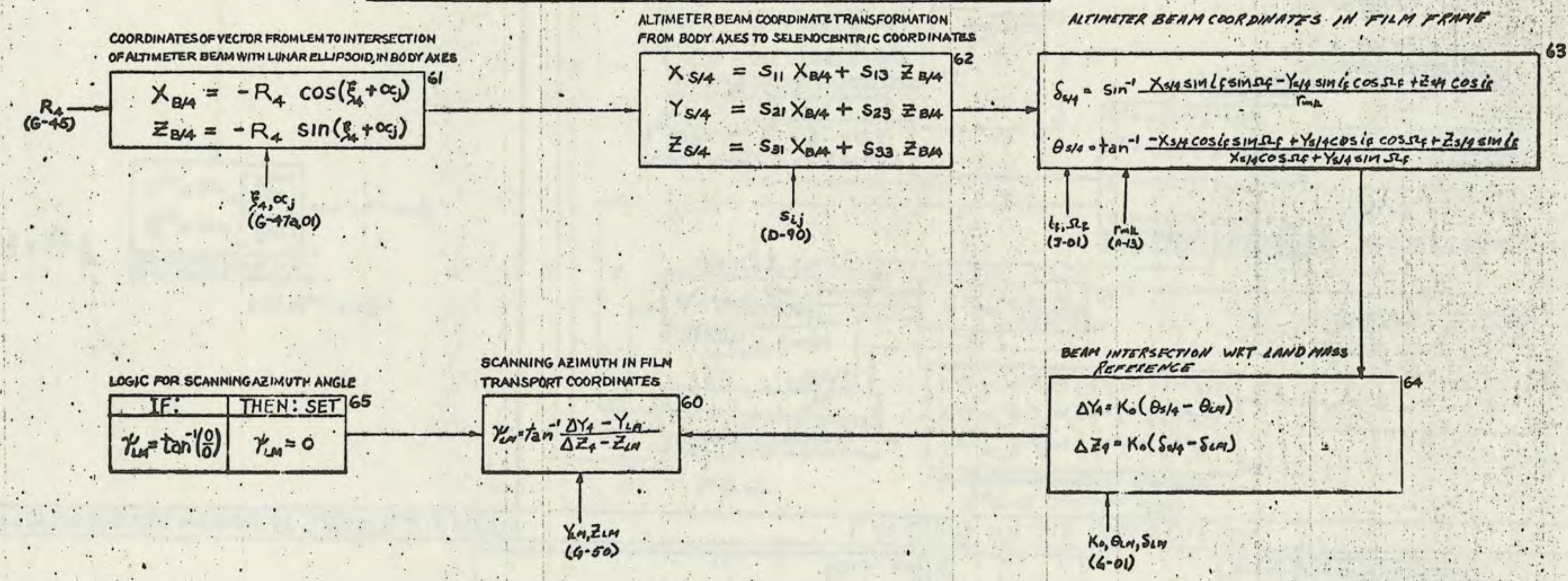
1668

G- LEM LUNAR LANDING RADAR G

LAND MASS SIMULATOR COORDINATE DRIVES



LAND MASS SIMULATOR AZIMUTH DRIVE

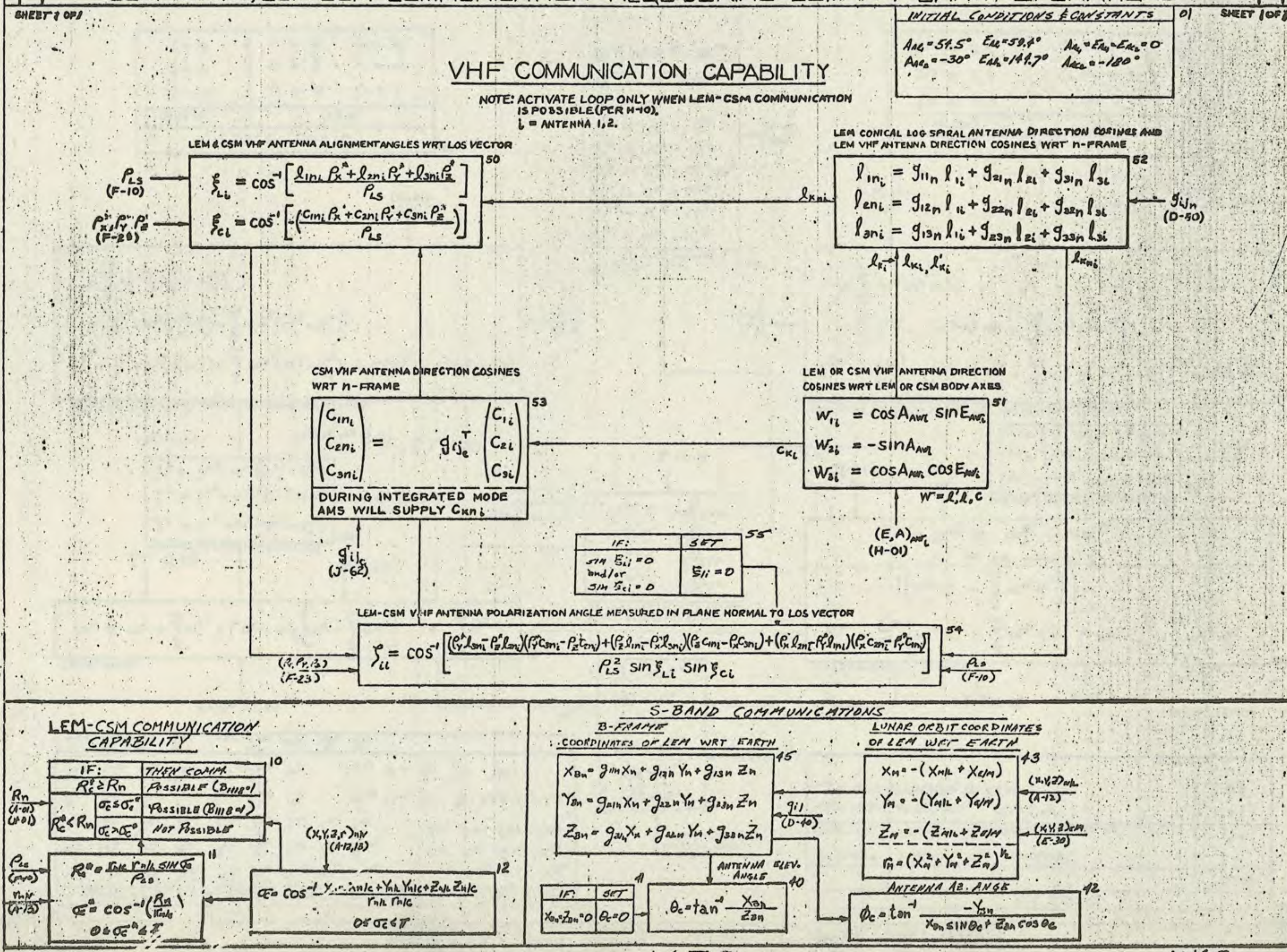


1114

H LEM-EARTH, LEM-CSM COMMUNICATION REQS. DURING LUNAR & EARTH OPERATIONS H

SHEET 2 OF 2

SHEET 1 OF 2



INITIAL CONDITIONS & CONSTANTS

$A_{AL_1} = 51.5°$ $E_{AL_1} = 59.4°$ $A_{AC_1} = E_{AC_1} = E_{AC_2} = 0$

$A_{AL_2} = -30°$ $E_{AL_2} = 141.7°$ $A_{AC_2} = -180°$

VHF COMMUNICATION CAPABILITY

NOTE: ACTIVATE LOOP ONLY WHEN LEM-CSM COMMUNICATION IS POSSIBLE (PER H-10).
i = ANTENNA 1, 2.

LEM & CSM VHF ANTENNA ALIGNMENT ANGLES WRT LOS VECTOR

$$\xi_{Li} = \cos^{-1}\left[\frac{l_{1ni}\rho_x^s + l_{2ni}\rho_y^s + l_{3ni}\rho_z^s}{\rho_{LS}}\right]$$

$$\xi_{ci} = \cos^{-1}\left[-\left(\frac{c_{1ni}\rho_x^s + c_{2ni}\rho_y^s + c_{3ni}\rho_z^s}{\rho_{LS}}\right)\right]$$

LEM CONICAL LOG SPIRAL ANTENNA DIRECTION COSINES AND LEM VHF ANTENNA DIRECTION COSINES WRT n-FRAME

$$l_{1ni} = g_{11n} l_{1i} + g_{21n} l_{2i} + g_{31n} l_{3i}$$

$$l_{2ni} = g_{12n} l_{1i} + g_{22n} l_{2i} + g_{32n} l_{3i}$$

$$l_{3ni} = g_{13n} l_{1i} + g_{23n} l_{2i} + g_{33n} l_{3i}$$

CSM VHF ANTENNA DIRECTION COSINES WRT n-FRAME

$$\begin{pmatrix} C_{1ni} \\ C_{2ni} \\ C_{3ni} \end{pmatrix} = g_{ij_c}^T \begin{pmatrix} C_{1i} \\ C_{2i} \\ C_{3i} \end{pmatrix}$$

DURING INTEGRATED MODE AMS WILL SUPPLY C_{kni}

LEM OR CSM VHF ANTENNA DIRECTION COSINES WRT LEM OR CSM BODY AXES

$$W_{1i} = \cos A_{AWi} \sin E_{AWi}$$

$$W_{2i} = -\sin A_{AWi}$$

$$W_{3i} = \cos A_{AWi} \cos E_{AWi}$$

$W = l, l', c$

IF:	SET
$\sin \xi_{Li} = 0$ and/or $\sin \xi_{ci} = 0$	$\zeta_{li} = 0$

LEM-CSM VHF ANTENNA POLARIZATION ANGLE MEASURED IN PLANE NORMAL TO LOS VECTOR

$$\zeta_{li} = \cos^{-1}\left[\frac{(\rho_y^s l_{3ni} - \rho_z^s l_{2ni})(\rho_y^s c_{3ni} - \rho_z^s c_{2ni}) + (\rho_z^s l_{1ni} - \rho_x^s l_{3ni})(\rho_z^s c_{1ni} - \rho_x^s c_{3ni}) + (\rho_x^s l_{2ni} - \rho_y^s l_{1ni})(\rho_x^s c_{2ni} - \rho_y^s c_{1ni})}{\rho_{LS}^2 \sin\xi_{Li} \sin\xi_{ci}}\right]$$

LEM-CSM COMMUNICATION CAPABILITY

IF:		THEN COMM
$R_c^* \ge R_n$		POSSIBLE ($B_{III8} = 1$)
$R_c^* < R_n$	$\sigma_c \le \sigma_c^*$	POSSIBLE ($B_{III8} = 1$)
	$\sigma_c > \sigma_c^*$	NOT POSSIBLE

$$R_c^* = \frac{r_{nlc}\, r_{nl} \sin\sigma_c}{\rho_{LS}}$$

$$\sigma_c^* = \cos^{-1}\left(\frac{R_n}{r_{nlc}}\right)$$

$$0 \le \sigma_c^* \le \pi$$

$$\sigma_c = \cos^{-1}\frac{X_{nl}X_{nlc} + Y_{nl}Y_{nlc} + Z_{nl}Z_{nlc}}{r_{nl}\, r_{nlc}}$$

$$0 \le \sigma_c \le \pi$$

S-BAND COMMUNICATIONS

B-FRAME COORDINATES OF LEM WRT EARTH

$$X_{Bn} = g_{11n} X_n + g_{12n} Y_n + g_{13n} Z_n$$

$$Y_{Bn} = g_{21n} X_n + g_{22n} Y_n + g_{23n} Z_n$$

$$Z_{Bn} = g_{31n} X_n + g_{32n} Y_n + g_{33n} Z_n$$

LUNAR ORBIT COORDINATES OF LEM WRT EARTH

$$X_M = -(X_{MIL} + X_{E/M})$$

$$Y_M = -(Y_{MIL} + Y_{E/M})$$

$$Z_M = -(Z_{MIL} + Z_{E/M})$$

$$r_M = (X_M^2 + Y_M^2 + Z_M^2)^{1/2}$$

ANTENNA ELEV. ANGLE

$$\theta_c = \tan^{-1}\frac{X_{Bn}}{Z_{Bn}}$$

IF:	SET
$X_{Bn} = Z_{Bn} = 0$	$\theta_c = 0$

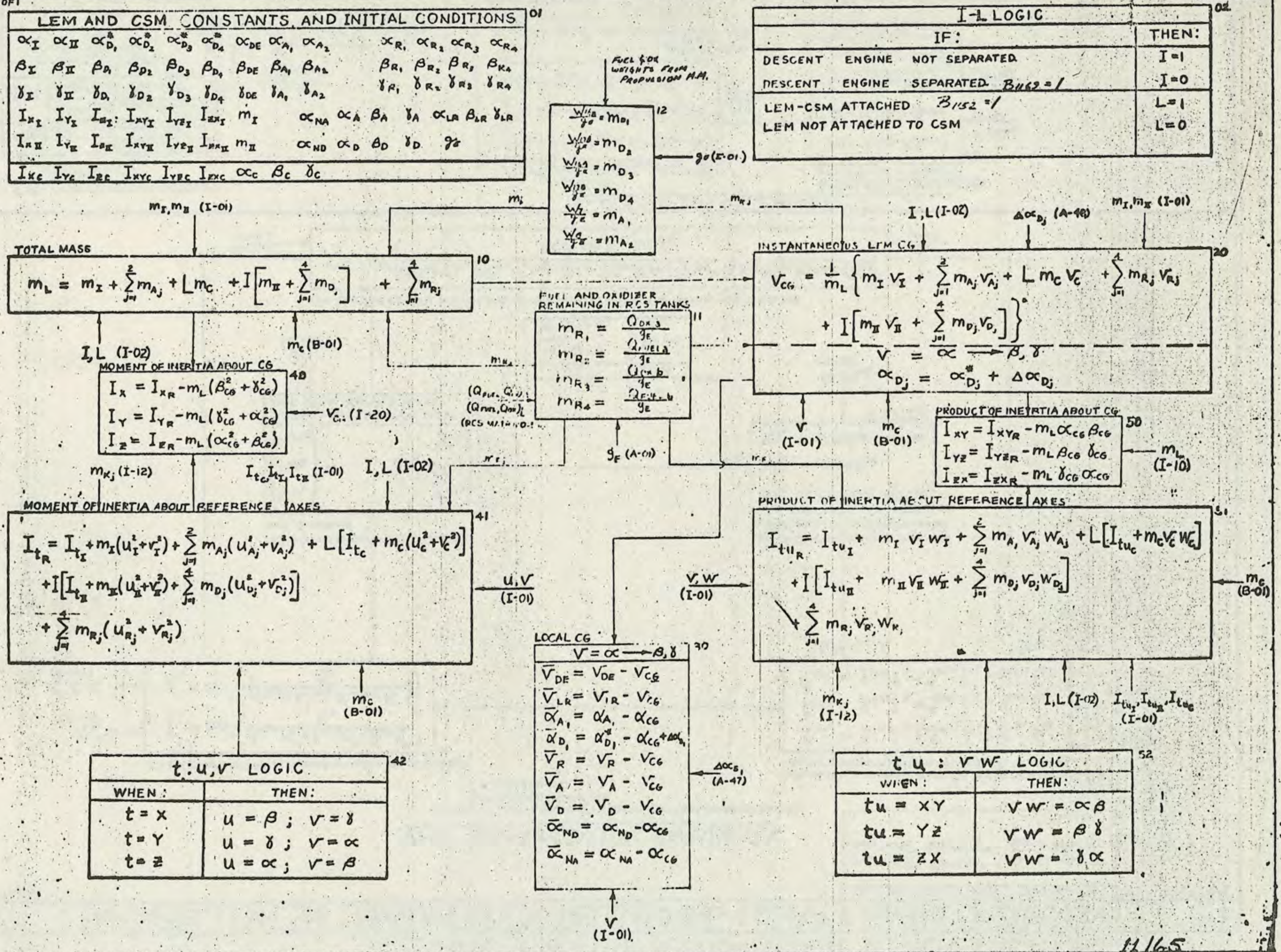
ANTENNA AZ. ANGLE

$$\phi_c = \tan^{-1}\frac{-Y_{Bn}}{X_{Bn}\sin\theta_c + Z_{Bn}\cos\theta_c}$$

115

I- WEIGHTS AND BALANCE I

SHEET 1 OF 1



LEM AND CSM CONSTANTS AND INITIAL CONDITIONS 01

$\alpha_{\mathrm{I}}\ \alpha_{\mathrm{II}}\ \alpha^*_{D_1}\ \alpha^*_{D_2}\ \alpha^*_{D_3}\ \alpha^*_{D_4}\ \alpha_{DE}\ \alpha_{A_1}\ \alpha_{A_2}\quad \alpha_{R_1}\ \alpha_{R_2}\ \alpha_{R_3}\ \alpha_{R_4}$

$\beta_{\mathrm{I}}\ \beta_{\mathrm{II}}\ \beta_{D_1}\ \beta_{D_2}\ \beta_{D_3}\ \beta_{D_4}\ \beta_{DE}\ \beta_{A_1}\ \beta_{A_2}\quad \beta_{R_1}\ \beta_{R_2}\ \beta_{R_3}\ \beta_{R_4}$

$\gamma_{\mathrm{I}}\ \gamma_{\mathrm{II}}\ \gamma_{D_1}\ \gamma_{D_2}\ \gamma_{D_3}\ \gamma_{D_4}\ \gamma_{DE}\ \gamma_{A_1}\ \gamma_{A_2}\quad \gamma_{R_1}\ \gamma_{R_2}\ \gamma_{R_3}\ \gamma_{R_4}$

$I_{X_{\mathrm{I}}}\ I_{Y_{\mathrm{I}}}\ I_{Z_{\mathrm{I}}}\ I_{XY_{\mathrm{I}}}\ I_{YZ_{\mathrm{I}}}\ I_{ZX_{\mathrm{I}}}\ m_{\mathrm{I}}\quad \alpha_{NA}\ \alpha_A\ \beta_A\ \gamma_A\ \alpha_{LR}\ \beta_{LR}\ \gamma_{LR}$

$I_{X_{\mathrm{II}}}\ I_{Y_{\mathrm{II}}}\ I_{Z_{\mathrm{II}}}\ I_{XY_{\mathrm{II}}}\ I_{YZ_{\mathrm{II}}}\ I_{ZX_{\mathrm{II}}}\ m_{\mathrm{II}}\quad \alpha_{ND}\ \alpha_D\ \beta_D\ \gamma_D\ g_E$

$I_{X_C}\ I_{Y_C}\ I_{Z_C}\ I_{XY_C}\ I_{YZ_C}\ I_{ZX_C}\ \alpha_C\ \beta_C\ \gamma_C$

I-L LOGIC 02

IF:	THEN:
DESCENT ENGINE NOT SEPARATED	I=1
DESCENT ENGINE SEPARATED $B_{1169}=1$	I=0
LEM-CSM ATTACHED $B_{1152}=1$	L=1
LEM NOT ATTACHED TO CSM	L=0

12

$$\frac{W_{1118}}{g_0} = m_{D_1}\qquad \frac{W_{1118}}{g_E} = m_{D_2}\qquad \frac{W_{1124}}{g_E} = m_{D_3}\qquad \frac{W_{1120}}{g_E} = m_{D_4}\qquad \frac{W_7}{g_E} = m_{A_1}\qquad \frac{W_9}{g_E} = m_{A_2}$$

TOTAL MASS 10

$$m_L = m_{\mathrm{I}} + \sum_{j=1}^{2} m_{A_j} + L m_C + \mathrm{I}\left[m_{\mathrm{II}} + \sum_{j=1}^{4} m_{D_j}\right] + \sum_{j=1}^{4} m_{R_j}$$

FUEL AND OXIDIZER REMAINING IN RCS TANKS 11

$$m_{R_1} = \frac{Q_{OX_a}}{g_E}\qquad m_{R_2} = \frac{Q_{FUEL_a}}{g_E}\qquad m_{R_3} = \frac{Q_{OX_b}}{g_E}\qquad m_{R_4} = \frac{Q_{FUEL_b}}{g_E}$$

INSTANTANEOUS LEM CG 20

$$V_{CG} = \frac{1}{m_L}\left\{m_{\mathrm{I}} V_{\mathrm{I}} + \sum_{j=1}^{2} m_{A_j} V_{A_j} + L m_C V_C + \sum_{j=1}^{4} m_{R_j} V_{R_j} + \mathrm{I}\left[m_{\mathrm{II}} V_{\mathrm{II}} + \sum_{j=1}^{4} m_{D_j} V_{D_j}\right]\right\}$$

$$V = \alpha \longrightarrow \beta, \gamma$$

$$\alpha_{D_j} = \alpha^*_{D_j} + \Delta\alpha_{D_j}$$

MOMENT OF INERTIA ABOUT CG 40

$$I_X = I_{X_R} - m_L(\beta^2_{CG} + \gamma^2_{CG})$$
$$I_Y = I_{Y_R} - m_L(\gamma^2_{CG} + \alpha^2_{CG})$$
$$I_Z = I_{Z_R} - m_L(\alpha^2_{CG} + \beta^2_{CG})$$

PRODUCT OF INERTIA ABOUT CG 50

$$I_{XY} = I_{XY_R} - m_L \alpha_{CG} \beta_{CG}$$
$$I_{YZ} = I_{YZ_R} - m_L \beta_{CG} \gamma_{CG}$$
$$I_{ZX} = I_{ZX_R} - m_L \gamma_{CG} \alpha_{CG}$$

MOMENT OF INERTIA ABOUT REFERENCE AXES 41

$$I_{t_R} = I_{t_{\mathrm{I}}} + m_{\mathrm{I}}(u^2_{\mathrm{I}} + v^2_{\mathrm{I}}) + \sum_{j=1}^{2} m_{A_j}(u^2_{A_j} + v^2_{A_j}) + L\left[I_{t_C} + m_C(u^2_C + v^2_C)\right] + \mathrm{I}\left[I_{t_{\mathrm{II}}} + m_{\mathrm{II}}(u^2_{\mathrm{II}} + v^2_{\mathrm{II}}) + \sum_{j=1}^{4} m_{D_j}(u^2_{D_j} + v^2_{D_j})\right] + \sum_{j=1}^{4} m_{R_j}(u^2_{R_j} + v^2_{R_j})$$

PRODUCT OF INERTIA ABOUT REFERENCE AXES 51

$$I_{tu_R} = I_{tu_{\mathrm{I}}} + m_{\mathrm{I}} v_{\mathrm{I}} w_{\mathrm{I}} + \sum_{j=1}^{2} m_{A_j} v_{A_j} w_{A_j} + L\left[I_{tu_C} + m_C v_C w_C\right] + \mathrm{I}\left[I_{tu_{\mathrm{II}}} + m_{\mathrm{II}} v_{\mathrm{II}} w_{\mathrm{II}} + \sum_{j=1}^{4} m_{D_j} v_{D_j} w_{D_j}\right] + \sum_{j=1}^{4} m_{R_j} v_{R_j} w_{R_j}$$

LOCAL CG 30

$$V = \alpha \longrightarrow \beta, \gamma$$

$$\bar{V}_{DE} = V_{DE} - V_{CG}$$
$$\bar{V}_{LR} = V_{LR} - V_{CG}$$
$$\bar{\alpha}_{A_1} = \alpha_{A_1} - \alpha_{CG}$$
$$\bar{\alpha}_{D_1} = \alpha^*_{D_1} - \alpha_{CG} + \Delta\alpha_{D_1}$$
$$\bar{V}_R = V_R - V_{CG}$$
$$\bar{V}_A = V_A - V_{CG}$$
$$\bar{V}_D = V_D - V_{CG}$$
$$\bar{\alpha}_{ND} = \alpha_{ND} - \alpha_{CG}$$
$$\bar{\alpha}_{NA} = \alpha_{NA} - \alpha_{CG}$$

t : u, v LOGIC 42

WHEN:	THEN:
t = X	$u = \beta$; $v = \gamma$
t = Y	$u = \gamma$; $v = \alpha$
t = Z	$u = \alpha$; $v = \beta$

t u : v w LOGIC 52

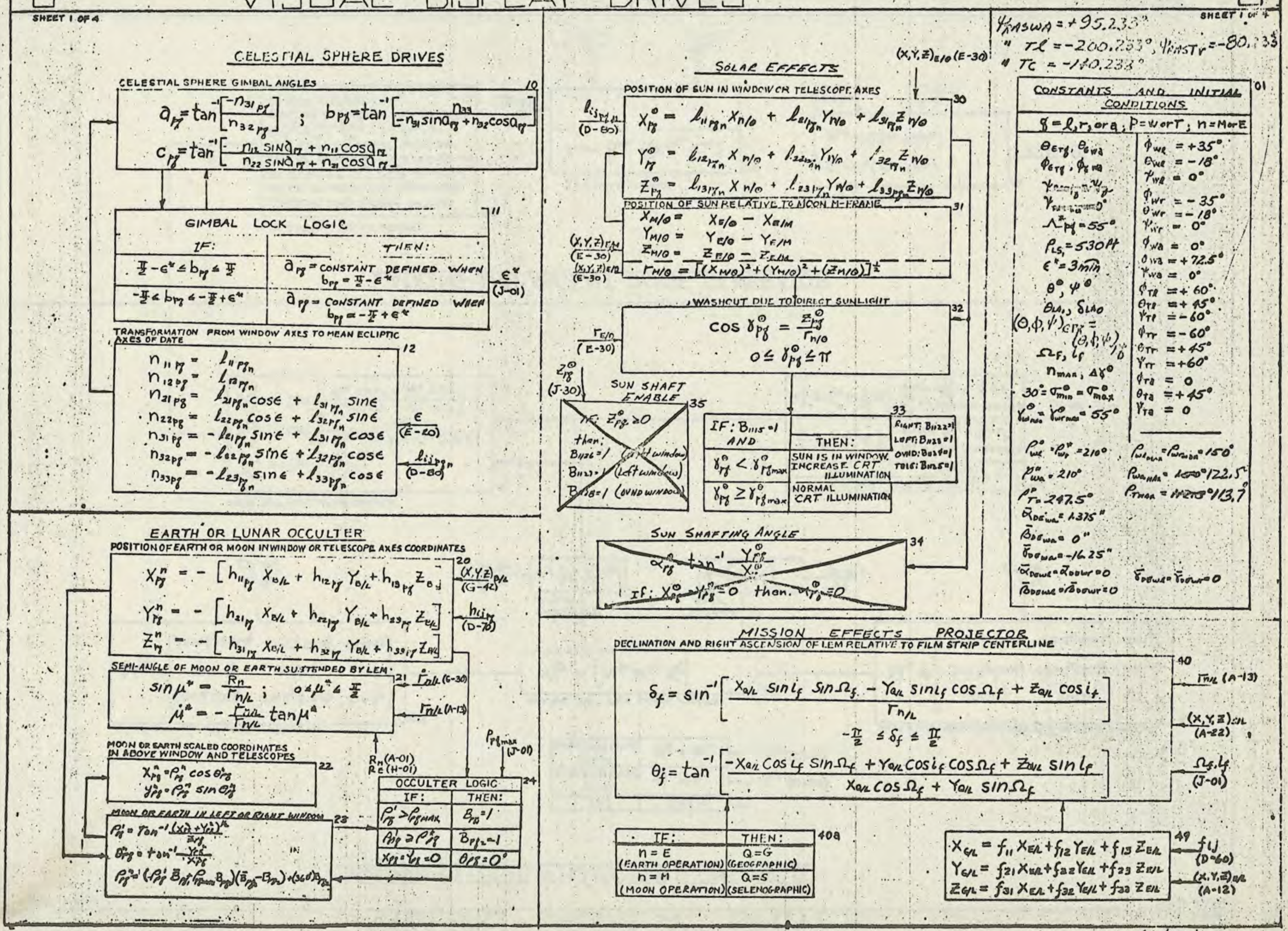
WHEN:	THEN:
tu = XY	$vw = \alpha\beta$
tu = YZ	$vw = \beta\gamma$
tu = ZX	$vw = \gamma\alpha$

11/65

1671

J- VISUAL DISPLAY DRIVES

SHEET 1 OF 4



CELESTIAL SPHERE DRIVES

CELESTIAL SPHERE GIMBAL ANGLES — 10

$$a_{p\gamma}=\tan^{-1}\left[\frac{-n_{31p\gamma}}{n_{32p\gamma}}\right]\;;\quad b_{p\gamma}=\tan^{-1}\left[\frac{n_{33}}{-n_{31}\sin a_{p\gamma}+n_{32}\cos a_{p\gamma}}\right]$$

$$c_{p\gamma}=\tan^{-1}\left[\frac{-n_{12}\sin a_{p\gamma}+n_{11}\cos a_{p\gamma}}{n_{22}\sin a_{p\gamma}+n_{21}\cos a_{p\gamma}}\right]$$

GIMBAL LOCK LOGIC — 11

IF:	THEN:
$\frac{\pi}{2}-\epsilon^{*}\le b_{p\gamma}\le\frac{\pi}{2}$	$a_{p\gamma}$ = CONSTANT DEFINED WHEN $b_{p\gamma}=\frac{\pi}{2}-\epsilon^{*}$
$-\frac{\pi}{2}\le b_{p\gamma}\le-\frac{\pi}{2}+\epsilon^{*}$	$a_{p\gamma}$ = CONSTANT DEFINED WHEN $b_{p\gamma}=-\frac{\pi}{2}+\epsilon^{*}$

ϵ^{*} (J-01)

TRANSFORMATION FROM WINDOW AXES TO MEAN ECLIPTIC AXES OF DATE — 12

$$\begin{aligned}
n_{11p\gamma}&=l_{11p\gamma_n}\\
n_{12p\gamma}&=l_{12p\gamma_n}\\
n_{21p\gamma}&=l_{21p\gamma_n}\cos\epsilon+l_{31p\gamma_n}\sin\epsilon\\
n_{22p\gamma}&=l_{22p\gamma_n}\cos\epsilon+l_{32p\gamma_n}\sin\epsilon\\
n_{31p\gamma}&=-l_{21p\gamma_n}\sin\epsilon+l_{31p\gamma_n}\cos\epsilon\\
n_{32p\gamma}&=-l_{22p\gamma_n}\sin\epsilon+l_{32p\gamma_n}\cos\epsilon\\
n_{33p\gamma}&=-l_{23p\gamma_n}\sin\epsilon+l_{33p\gamma_n}\cos\epsilon
\end{aligned}$$

ϵ (E-20)

$l_{ijp\gamma_n}$ (D-80)

EARTH OR LUNAR OCCULTER

POSITION OF EARTH OR MOON IN WINDOW OR TELESCOPE AXES COORDINATES — 20

$$\begin{aligned}
X^{n}_{p\gamma}&=-\left[h_{11p\gamma}X_{B/L}+h_{12p\gamma}Y_{B/L}+h_{13p\gamma}Z_{B/L}\right]\\
Y^{n}_{p\gamma}&=-\left[h_{21p\gamma}X_{B/L}+h_{22p\gamma}Y_{B/L}+h_{23p\gamma}Z_{B/L}\right]\\
Z^{n}_{p\gamma}&=-\left[h_{31p\gamma}X_{B/L}+h_{32p\gamma}Y_{B/L}+h_{33p\gamma}Z_{B/L}\right]
\end{aligned}$$

$(X,Y,Z)_{B/L}$ (G-42)

$h_{ijp\gamma}$ (D-70)

SEMI-ANGLE OF MOON OR EARTH SUSTENDED BY LEM — 21

$$\sin\mu^{*}=\frac{R_n}{r_{n/L}}\;;\quad 0\le\mu^{*}\le\frac{\pi}{2}$$

$$\dot{\mu}^{*}=-\frac{\dot{r}_{n/L}}{r_{n/L}}\tan\mu^{*}$$

$\dot{r}_{n/L}$ (G-30)

$r_{n/L}$ (A-13)

R_M (A-01)
R_E (H-01)

MOON OR EARTH SCALED COORDINATES IN ABOVE WINDOW AND TELESCOPES — 22

$$X^{n}_{p\gamma}=\rho^{n}_{p\gamma}\cos\theta^{n}_{p\gamma}$$

$$Y^{n}_{p\gamma}=\rho^{n}_{p\gamma}\sin\theta^{n}_{p\gamma}$$

MOON OR EARTH IN LEFT OR RIGHT WINDOW — 23

$$\rho^{1}_{p\gamma}=\tan^{-1}\frac{(X^{2}_{p\gamma}+Y^{2}_{p\gamma})^{1/2}}{Z_{p\gamma}}$$

$$\theta^{n}_{p\gamma}=\tan^{-1}\frac{Y^{n}_{p\gamma}}{X^{n}_{p\gamma}}$$

$$\rho^{n}_{p\gamma}=(\rho^{1}_{p\gamma}\bar{B}_{p\gamma_1}+\rho_{p\gamma\,max}B_{p\gamma_1})(\bar{B}_{p\gamma_2}-B_{p\gamma_2})+(360)B_{p\gamma_2}$$

$\rho_{p\gamma\,max}$ (J-01)

OCCULTER LOGIC — 24

IF:	THEN:
$\rho^{1}_{p\gamma}>\rho_{p\gamma\,MAX}$	$B_{p\gamma_1}=1$
$\mu^{*}_{p\gamma}\ge\rho^{1}_{p\gamma}$	$B_{p\gamma_2}=1$
$X_{p\gamma}=Y_{p\gamma}=0$	$\theta_{p\gamma}=0°$

SOLAR EFFECTS

$(X,Y,Z)_{E/\odot}$ (E-30)

POSITION OF SUN IN WINDOW OR TELESCOPE AXES — 30

$$\begin{aligned}
X^{\odot}_{p\gamma}&=l_{11p\gamma_n}X_{M/\odot}+l_{21p\gamma_n}Y_{M/\odot}+l_{31p\gamma_n}Z_{M/\odot}\\
Y^{\odot}_{p\gamma}&=l_{12p\gamma_n}X_{M/\odot}+l_{22p\gamma_n}Y_{M/\odot}+l_{32p\gamma_n}Z_{M/\odot}\\
Z^{\odot}_{p\gamma}&=l_{13p\gamma_n}X_{M/\odot}+l_{23p\gamma_n}Y_{M/\odot}+l_{33p\gamma_n}Z_{M/\odot}
\end{aligned}$$

$l_{ijp\gamma_n}$ (D-80)

POSITION OF SUN RELATIVE TO MOON M-FRAME — 31

$$\begin{aligned}
X_{M/\odot}&=X_{E/\odot}-X_{E/M}\\
Y_{M/\odot}&=Y_{E/\odot}-Y_{E/M}\\
Z_{M/\odot}&=Z_{E/\odot}-Z_{E/M}\\
r_{M/\odot}&=\left[(X_{M/\odot})^{2}+(Y_{M/\odot})^{2}+(Z_{M/\odot})^{2}\right]^{\frac{1}{2}}
\end{aligned}$$

$(X,Y,Z)_{E/M}$ (E-30)

$(X,Y,Z)_{E/\odot}$ (E-30)

WASHOUT DUE TO DIRECT SUNLIGHT — 32

$$\cos\gamma^{\odot}_{p\gamma}=\frac{Z^{\odot}_{p\gamma}}{r_{M/\odot}}$$

$$0\le\gamma^{\odot}_{p\gamma}\le\pi$$

$r_{E/\odot}$ (E-30)

$Z^{\odot}_{p\gamma}$ (J-30)

SUN SHAFT ENABLE — 35

~~IF: $Z^{\odot}_{p\gamma}\ge0$~~

~~then:~~

~~$B_{1126}=1$ (right window)~~

~~$B_{1127}=1$ (Left window)~~

~~$B_{1128}=1$ (OVHD WINDOW)~~

33

IF: $B_{1115}=1$ AND	THEN:	RIGHT: $B_{1122}=1$ LEFT: $B_{1123}=1$ OVHD: $B_{1124}=1$ TELE: $B_{1125}=1$
$\gamma^{\odot}_{p\gamma}<\gamma^{\odot}_{p\gamma\,max}$	SUN IS IN WINDOW. INCREASE CRT ILLUMINATION	
$\gamma^{\odot}_{p\gamma}\ge\gamma^{\odot}_{p\gamma\,max}$	NORMAL CRT ILLUMINATION	

SUN SHAFTING ANGLE — 34

~~$\alpha^{\odot}_{p\gamma}=\tan^{-1}\frac{Y^{\odot}_{p\gamma}}{X^{\odot}_{p\gamma}}$~~

~~IF: $X^{\odot}_{p\gamma}=Y^{\odot}_{p\gamma}=0$ then. $\alpha^{\odot}_{p\gamma}=0$~~

$\psi_{\wedge ASWA}=+95.233°$

" $T\ell=-200.233°$, $\psi_{\wedge ASTr}=-80.233$

" $TC=-140.233°$

CONSTANTS AND INITIAL CONDITIONS — 01

$\gamma=l,r,$ or a; $P=W$ or T; $n=M$ or E

$\theta_{ET\gamma},\theta_{GWa}$	$\phi_{Wl}=+35°$
$\phi_{ET\gamma},\phi_{GWa}$	$\theta_{Wl}=-18°$
$\psi_{[illegible]}=\psi_{\gamma}$	$\psi_{Wl}=0°$
$\psi_{[illegible]}=0°$	$\phi_{Wr}=-35°$
$\Lambda^{*}_{p\gamma}=55°$	$\theta_{Wr}=-18°$
$\rho_{LS_1}=530$ ft	$\psi_{Wr}=0°$
$\epsilon^{*}=3$ min	$\phi_{Wa}=0°$
$\theta^{\odot},\psi^{\odot}$	$\theta_{Wa}=+72.5°$
$\theta_{LAO},\delta_{LAO}$	$\psi_{Wa}=0°$
$(\Theta,\Phi,\Psi)_{ET\gamma}=(\theta,\phi,\psi)_{\gamma_0}$	$\phi_{Tl}=+60°$
	$\theta_{Tl}=+45°$
Ω_F,i_f	$\psi_{Tl}=-60°$
$n_{max},\Delta\gamma^{\odot}$	$\phi_{Tr}=-60°$
$30°=\sigma^{\odot}_{min}=\sigma^{\odot}_{max}$	$\theta_{Tr}=+45°$
$\gamma^{\odot}_{Wl\,max}=\gamma^{\odot}_{Wr\,max}=55°$	$\psi_{Tr}=+60°$
	$\phi_{Ta}=0$
	$\theta_{Ta}=+45°$
	$\psi_{Ta}=0$

$\rho^{p}_{Wl}=\rho^{p}_{Wr}=210°$, $\rho_{Wl\,max}=\rho_{Wr\,max}=150°$

$\rho^{p}_{Wa}=210°$ $\rho_{Wa\,max}=$ ~~150~~$°$ $122.5°$

$\rho^{p}_{T}=247.5°$ $\rho_{T\,max}=$ ~~[illegible]~~$°$ $113.7°$

$\bar{\alpha}_{DEWa}=1.375''$

$\bar{\beta}_{DEWa}=0''$

$\bar{\gamma}_{DEWa}=-16.25''$

$\bar{\alpha}_{DEWl}=\bar{\alpha}_{DEWr}=0$ $\bar{\gamma}_{DEWl}=\bar{\gamma}_{DEWr}=0$

$\bar{\beta}_{DEWl}=\bar{\beta}_{DEWr}=0$

SHEET 1 OF 4

MISSION EFFECTS PROJECTOR

DECLINATION AND RIGHT ASCENSION OF LEM RELATIVE TO FILM STRIP CENTERLINE — 40

$$\delta_f=\sin^{-1}\left[\frac{X_{Q/L}\sin i_f\sin\Omega_f-Y_{Q/L}\sin i_f\cos\Omega_f+Z_{Q/L}\cos i_f}{r_{n/L}}\right]$$

$$-\frac{\pi}{2}\le\delta_f\le\frac{\pi}{2}$$

$$\theta_f=\tan^{-1}\left[\frac{-X_{Q/L}\cos i_f\sin\Omega_f+Y_{Q/L}\cos i_f\cos\Omega_f+Z_{Q/L}\sin i_f}{X_{Q/L}\cos\Omega_f+Y_{Q/L}\sin\Omega_f}\right]$$

$r_{n/L}$ (A-13)

$(X,Y,Z)_{S/L}$ (A-22)

Ω_f,i_f (J-01)

40a

IF:	THEN:
$n=E$ (EARTH OPERATION)	$Q=G$ (GEOGRAPHIC)
$n=M$ (MOON OPERATION)	$Q=S$ (SELENOGRAPHIC)

49

$$\begin{aligned}
X_{G/L}&=f_{11}X_{E/L}+f_{12}Y_{E/L}+f_{13}Z_{E/L}\\
Y_{G/L}&=f_{21}X_{E/L}+f_{22}Y_{E/L}+f_{23}Z_{E/L}\\
Z_{G/L}&=f_{31}X_{E/L}+f_{32}Y_{E/L}+f_{33}Z_{E/L}
\end{aligned}$$

f_{ij} (D-60)

$(X,Y,Z)_{E/L}$ (A-12)

J

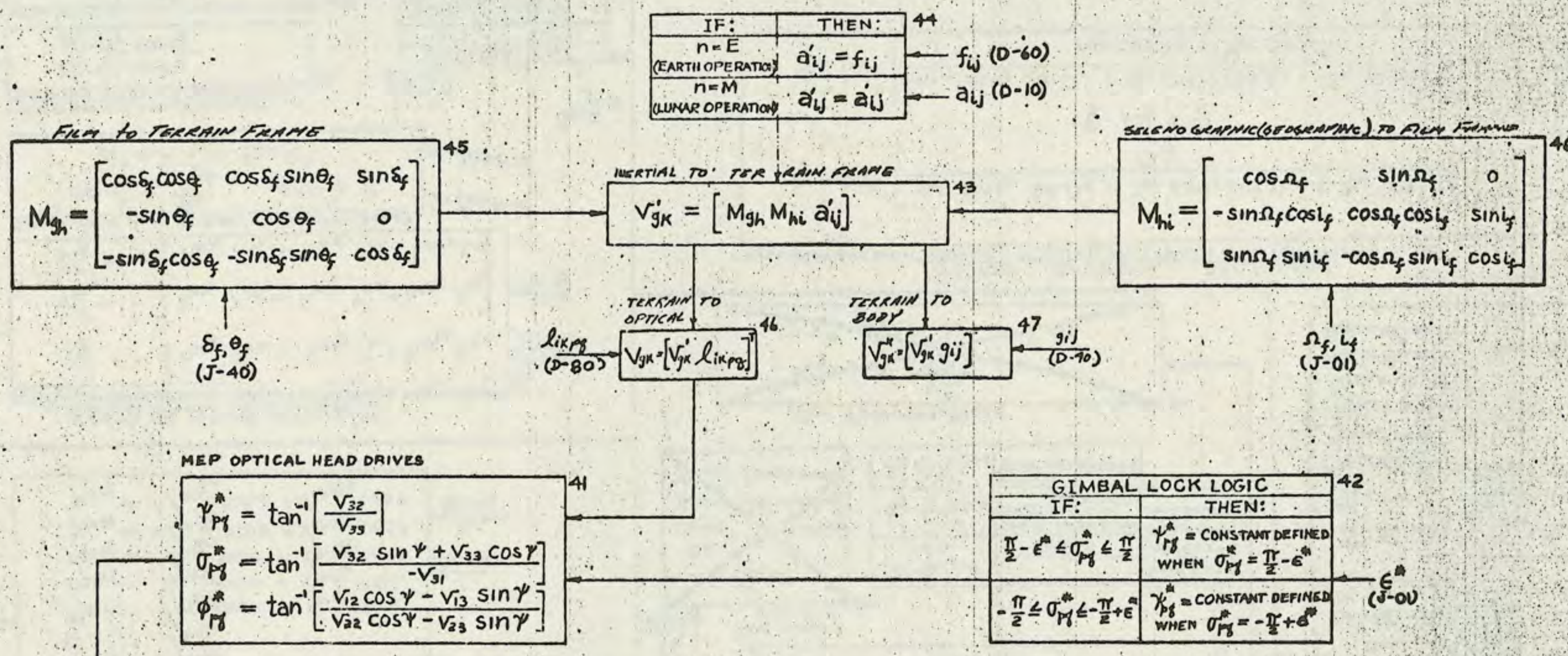
VISUAL DISPLAY DRIVES

J

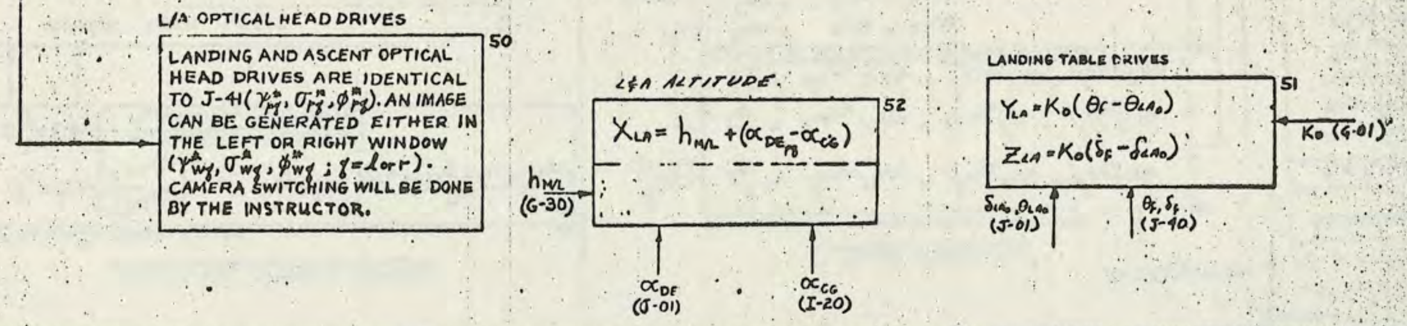
SHEET 2 OF 4

SHEET 2 OF 4

MISSION EFFECTS PROJECTOR (CONTINUED)

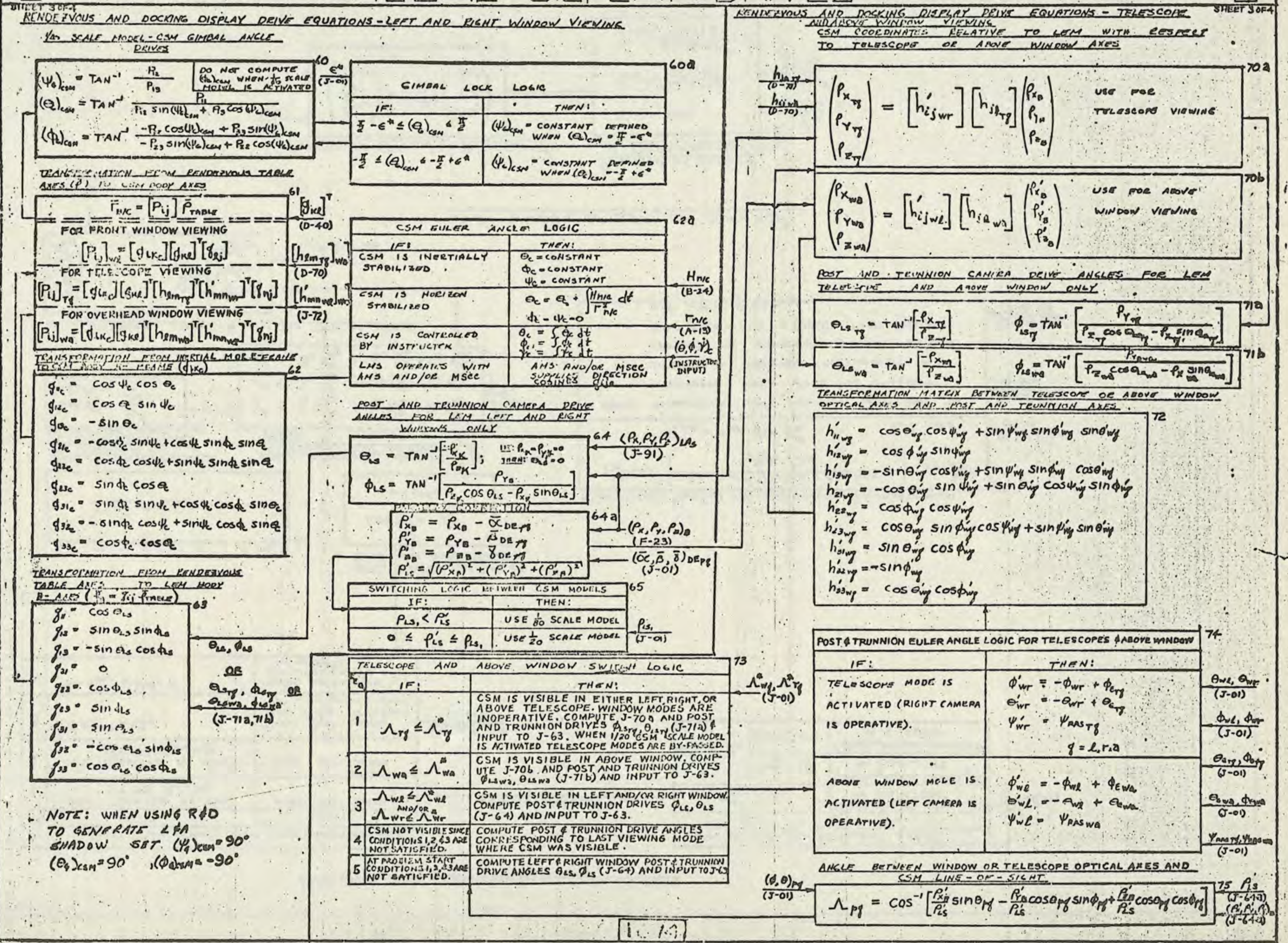


LANDING AND ASCENT IMAGE GENERATOR



LANDING AND ASCENT OPTICAL HEAD DRIVES ARE IDENTICAL TO J-41($\psi^*_{p\gamma}, \sigma^*_{p\gamma}, \phi^*_{p\gamma}$). AN IMAGE CAN BE GENERATED EITHER IN THE LEFT OR RIGHT WINDOW ($\psi^*_{w\gamma}, \sigma^*_{w\gamma}, \phi^*_{w\gamma}$; $\gamma = l$ or r). CAMERA SWITCHING WILL BE DONE BY THE INSTRUCTOR.

VISUAL DISPLAY DRIVES



RENDEZVOUS AND DOCKING DISPLAY DRIVE EQUATIONS - LEFT AND RIGHT WINDOW VIEWING

1/80 SCALE MODEL - CSM GIMBAL ANGLE DRIVES

60

$$(\psi_G)_{CSM} = \tan^{-1}\frac{P_{12}}{P_{13}}$$

DO NOT COMPUTE $(\psi_G)_{CSM}$ WHEN $\frac{1}{20}$ SCALE MODEL IS ACTIVATED

$$(\theta_G)_{CSM} = \tan^{-1}\frac{P_{11}}{P_{12}\sin(\psi_G)_{CSM} + P_{13}\cos(\psi_G)_{CSM}}$$

$$(\phi_G)_{CSM} = \tan^{-1}\frac{-P_{32}\cos(\psi_G)_{CSM} + P_{33}\sin(\psi_G)_{CSM}}{-P_{23}\sin(\psi_G)_{CSM} + P_{22}\cos(\psi_G)_{CSM}}$$

60a

GIMBAL LOCK LOGIC	
IF:	THEN:
$\frac{\pi}{2} - \epsilon^* \le (\theta_G)_{CSM} < \frac{\pi}{2}$	$(\psi_G)_{CSM}$ = CONSTANT DEFINED WHEN $(\theta_G)_{CSM} = \frac{\pi}{2} - \epsilon^*$
$-\frac{\pi}{2} \le (\theta_G)_{CSM} < -\frac{\pi}{2} + \epsilon^*$	$(\psi_G)_{CSM}$ = CONSTANT DEFINED WHEN $(\theta_G)_{CSM} = -\frac{\pi}{2} + \epsilon^*$

TRANSFORMATION FROM RENDEZVOUS TABLE AXES ($\bar P$) TO CSM BODY AXES

61

$$\bar r_{n/c} = [P_{ij}]\,\bar P_{TABLE}$$

FOR FRONT WINDOW VIEWING

$$[P_{ij}]_{wq} = [g_{\ell k_c}][g_{k\ell}]^T[\gamma_{\ell j}]$$

FOR TELESCOPE VIEWING

$$[P_{ij}]_{Tg} = [g_{\ell k_c}][g_{k\ell}]^T[h_{\ell m_{Tg}}]^T[h'_{mn_{wr}}]^T[\gamma_{nj}]$$

FOR OVERHEAD WINDOW VIEWING

$$[P_{ij}]_{wa} = [g_{\ell k_c}][g_{k\ell}]^T[h_{\ell m_{wa}}]^T[h'_{mn_{wq}}]^T[\gamma_{nj}]$$

TRANSFORMATION FROM INERTIAL MORE-FRAME TO CSM BODY AXES FRAME ($g_{\ell k_c}$)

62

$g_{11_c} = \cos\psi_c \cos\theta_c$

$g_{12_c} = \cos\theta_c \sin\psi_c$

$g_{13_c} = -\sin\theta_c$

$g_{21_c} = -\cos\phi_c \sin\psi_c + \cos\psi_c \sin\phi_c \sin\theta_c$

$g_{22_c} = \cos\phi_c \cos\psi_c + \sin\psi_c \sin\phi_c \sin\theta_c$

$g_{23_c} = \sin\phi_c \cos\theta_c$

$g_{31_c} = \sin\phi_c \sin\psi_c + \cos\psi_c \cos\phi_c \sin\theta_c$

$g_{32_c} = -\sin\phi_c \cos\psi_c + \sin\psi_c \cos\phi_c \sin\theta_c$

$g_{33_c} = \cos\phi_c \cos\theta_c$

62a

CSM EULER ANGLE LOGIC	
IF:	THEN:
CSM IS INERTIALLY STABILIZED.	θ_c = CONSTANT ϕ_c = CONSTANT ψ_c = CONSTANT
CSM IS HORIZON STABILIZED	$\theta_c = \theta_0 + \int \frac{H_{n/c}}{r^2_{n/c}}\,dt$ $\phi_c = \psi_c = 0$
CSM IS CONTROLLED BY INSTRUCTOR	$\theta_c = \int \dot\theta_c\,dt$ $\phi_c = \int \dot\phi_c\,dt$ $\psi_c = \int \dot\psi_c\,dt$
LMS OPERATES WITH AMS AND/OR MSCC	AMS AND/OR MSCC SUPPLIES DIRECTION COSINES $g_{\ell k_c}$

TRANSFORMATION FROM RENDEZVOUS TABLE AXES TO LEM BODY B-AXES ($\bar r_B = \gamma_{ij}\,\bar r_{TABLE}$)

63

$\gamma_{11} = \cos\theta_{LS}$

$\gamma_{12} = \sin\theta_{LS}\sin\phi_{LS}$

$\gamma_{13} = -\sin\theta_{LS}\cos\phi_{LS}$

$\gamma_{21} = 0$

$\gamma_{22} = \cos\phi_{LS}$

$\gamma_{23} = \sin\phi_{LS}$

$\gamma_{31} = \sin\theta_{LS}$

$\gamma_{32} = -\cos\theta_{LS}\sin\phi_{LS}$

$\gamma_{33} = \cos\theta_{LS}\cos\phi_{LS}$

NOTE: WHEN USING R&D TO GENERATE LPA SHADOW SET $(\psi_G)_{CSM} = 90°$ $(\theta_G)_{CSM} = 90°$, $(\phi_G)_{CSM} = -90°$

POST AND TRUNNION CAMERA DRIVE ANGLES FOR LEM LEFT AND RIGHT WINDOWS ONLY

64

$$\theta_{LS} = \tan^{-1}\left[\frac{-\rho_{x_K}}{\rho_{z_K}}\right]; \quad \text{IF: } \rho_{x_K} = \rho_{y_K} = 0 \text{ THEN: } \theta_{LS} = 0$$

$$\phi_{LS} = \tan^{-1}\left[\frac{\rho_{Y_B}}{\rho_{z_K}\cos\theta_{LS} - \rho_{x_K}\sin\theta_{LS}}\right]$$

[illegible]

64a

$$\rho'_{X_B} = \rho_{X_B} - \bar\alpha_{DE_{pg}}$$
$$\rho'_{Y_B} = \rho_{Y_B} - \bar\beta_{DE_{pg}}$$
$$\rho'_{Z_B} = \rho_{Z_B} - \bar\gamma_{DE_{pg}}$$
$$\rho'_{LS} = \sqrt{(\rho'_{X_B})^2 + (\rho'_{Y_B})^2 + (\rho'_{Z_B})^2}$$

65

SWITCHING LOGIC BETWEEN CSM MODELS	
IF:	THEN:
$\rho_{LS_1} < \rho'_{LS}$	USE $\frac{1}{80}$ SCALE MODEL
$0 \le \rho'_{LS} \le \rho_{LS_1}$	USE $\frac{1}{20}$ SCALE MODEL

73

TELESCOPE AND ABOVE WINDOW SWITCH LOGIC		
r_0	IF:	THEN:
1	$\Lambda_{Tg} \le \Lambda^*_{Tg}$	CSM IS VISIBLE IN EITHER LEFT, RIGHT, OR ABOVE TELESCOPE. WINDOW MODES ARE INOPERATIVE. COMPUTE J-70a AND POST AND TRUNNION DRIVES $\phi_{LS_{Tg}}, \theta_{LS_{Tg}}$ (J-71a) & INPUT TO J-63. WHEN 1/20 CSM SCALE MODEL IS ACTIVATED TELESCOPE MODES ARE BY-PASSED.
2	$\Lambda_{wa} \le \Lambda^*_{wa}$	CSM IS VISIBLE IN ABOVE WINDOW. COMPUTE J-70b AND POST AND TRUNNION DRIVES $\phi_{LSwa}, \theta_{LSwa}$ (J-71b) AND INPUT TO J-63.
3	$\Lambda_{w\ell} \le \Lambda^*_{w\ell}$ AND/OR $\Lambda_{wr} \le \Lambda^*_{wr}$	CSM IS VISIBLE IN LEFT AND/OR RIGHT WINDOW. COMPUTE POST & TRUNNION DRIVES ϕ_{LS}, θ_{LS} (J-64) AND INPUT TO J-63.
4	CSM NOT VISIBLE SINCE CONDITIONS 1, 2, & 3 ARE NOT SATISFIED.	COMPUTE POST & TRUNNION DRIVE ANGLES CORRESPONDING TO LAST VIEWING MODE WHERE CSM WAS VISIBLE.
5	AT PROBLEM START CONDITIONS 1, 2, & 3 ARE NOT SATISFIED.	COMPUTE LEFT & RIGHT WINDOW POST & TRUNNION DRIVE ANGLES θ_{LS}, ϕ_{LS} (J-64) AND INPUT TO J-63

RENDEZVOUS AND DOCKING DISPLAY DRIVE EQUATIONS - TELESCOPE AND ABOVE WINDOW VIEWING

CSM COORDINATES RELATIVE TO LEM WITH RESPECT TO TELESCOPE OR ABOVE WINDOW AXES

70a

$$\begin{pmatrix}\rho_{x_{Tg}}\\ \rho_{y_{Tg}}\\ \rho_{z_{Tg}}\end{pmatrix} = \left[h'_{ij_{wr}}\right]\left[h_{j\ell_{Tg}}\right]\begin{pmatrix}\rho_{x_B}\\ \rho_{y_B}\\ \rho_{z_B}\end{pmatrix}$$

USE FOR TELESCOPE VIEWING

70b

$$\begin{pmatrix}\rho_{x_{wa}}\\ \rho_{y_{wa}}\\ \rho_{z_{wa}}\end{pmatrix} = \left[h'_{ij_{w\ell}}\right]\left[h_{j\ell_{wa}}\right]\begin{pmatrix}\rho'_{x_B}\\ \rho'_{y_B}\\ \rho'_{z_B}\end{pmatrix}$$

USE FOR ABOVE WINDOW VIEWING

POST AND TRUNNION CAMERA DRIVE ANGLES FOR LEM TELESCOPE AND ABOVE WINDOW ONLY

71a

$$\theta_{LS_{Tg}} = \tan^{-1}\left[\frac{-\rho_{x_{Tg}}}{\rho_{z_{Tg}}}\right] \qquad \phi_{LS_{Tg}} = \tan^{-1}\left[\frac{\rho_{Y_{Tg}}}{\rho_{z_{Tg}}\cos\theta_{LS_{Tg}} - \rho_{x_{Tg}}\sin\theta_{LS_{Tg}}}\right]$$

71b

$$\theta_{LS_{wa}} = \tan^{-1}\left[\frac{-\rho_{x_{wa}}}{\rho_{z_{wa}}}\right] \qquad \phi_{LS_{wa}} = \tan^{-1}\left[\frac{\rho_{Y_{wa}}}{\rho_{z_{wa}}\cos\theta_{LS_{wa}} - \rho_{x_{wa}}\sin\theta_{LS_{wa}}}\right]$$

TRANSFORMATION MATRIX BETWEEN TELESCOPE OR ABOVE WINDOW OPTICAL AXES AND POST AND TRUNNION AXES

72

$h'_{11_{wq}} = \cos\theta'_{wq}\cos\psi'_{wq} + \sin\psi'_{wq}\sin\phi'_{wq}\sin\theta'_{wq}$

$h'_{12_{wq}} = \cos\phi'_{wq}\sin\psi'_{wq}$

$h'_{13_{wq}} = -\sin\theta'_{wq}\cos\psi'_{wq} + \sin\psi'_{wq}\sin\phi'_{wq}\cos\theta'_{wq}$

$h'_{21_{wq}} = -\cos\theta'_{wq}\sin\psi'_{wq} + \sin\theta'_{wq}\cos\psi'_{wq}\sin\phi'_{wq}$

$h'_{22_{wq}} = \cos\phi'_{wq}\cos\psi'_{wq}$

$h'_{23_{wq}} = \cos\theta'_{wq}\sin\phi'_{wq}\cos\psi'_{wq} + \sin\psi'_{wq}\sin\theta'_{wq}$

$h'_{31_{wq}} = \sin\theta'_{wq}\cos\phi'_{wq}$

$h'_{32_{wq}} = -\sin\phi'_{wq}$

$h'_{33_{wq}} = \cos\theta'_{wq}\cos\phi'_{wq}$

74

POST & TRUNNION EULER ANGLE LOGIC FOR TELESCOPES & ABOVE WINDOW	
IF:	THEN:
TELESCOPE MODE IS ACTIVATED (RIGHT CAMERA IS OPERATIVE).	$\phi'_{wr} = -\phi_{wr} + \phi_{eTg}$ $\theta'_{wr} = -\theta_{wr} + \theta_{eTg}$ $\psi'_{wr} = \psi_{RASTg}$ $q = \ell, r, a$
ABOVE WINDOW MODE IS ACTIVATED (LEFT CAMERA IS OPERATIVE).	$\phi'_{w\ell} = -\phi_{w\ell} + \phi_{Ewa}$ $\theta'_{w\ell} = -\theta_{w\ell} + \theta_{ewa}$ $\psi'_{w\ell} = \psi_{RASwa}$

ANGLE BETWEEN WINDOW OR TELESCOPE OPTICAL AXES AND CSM LINE-OF-SIGHT

75

$$\Lambda_{pg} = \cos^{-1}\left[\frac{\rho'_{x_B}}{\rho'_{LS}}\sin\theta_{pg} - \frac{\rho'_{Y_B}}{\rho'_{LS}}\cos\theta_{pg}\sin\phi_{pg} + \frac{\rho'_{z_B}}{\rho'_{LS}}\cos\theta_{pg}\cos\phi_{pg}\right]$$

11/65

J- VISUAL DISPLAY DRIVES J

LEM-CSM SUN ILLUMINATION REQUIREMENTS

80

	SOLAR SIMULATION LAMP LOGIC (1&2 OVERRIDE 4,5,6)	
	IF:	THEN:
1	SUN OCCULTED BY MOON (J-86) $B_{116}=0$	TURN OFF LAMPS
2	SUN OCCULTED BY EARTH (J-86) $B_{116}=0$	TURN OFF LAMPS
3	$0 \le \sigma^{\circ} \le \sigma_{min}$	TURN OFF LAMPS (SUN LIES IN FRONT OF LINE-OF-SIGHT)
4	$\pi - \sigma^{\circ}_{max} \le \sigma^{\circ} \le \pi$	TURN ON LAMPS (SUN LIES BEHIND LINE-OF-SIGHT)

$\sigma^{\circ}_{min}, \sigma^{\circ}_{max}, \sigma^{\circ}_{oo}$ (J-01)

CSM IN SHADOW

86

CSM- OR LEM-IN-SUNLIGHT LOGIC		
IF:		THEN:
$R^{*}_{n/0} \ge R_n$		CSM ($B_{116}=1$) OR LEM ($B_{115}=1$) IN SUNLIGHT
$R^{*}_{n/0} < R_n$	$0 \le \sigma_n^{\circ} \le \frac{\pi}{2}$	CSM($B_{116}=1$) OR LEM($B_{115}=1$) IN SUNLIGHT
	$\frac{\pi}{2} \le \sigma_n^{\circ} \le \pi$	CSM($B_{116}=0$) OR LEM ($B_{115}=0$) IN SHADOW

R_n, R_e (A-01)(H-01)

ANGLE BETWEEN LEM LINE OF SIGHT AND SUN'S DIRECTION

82

$$\cos\sigma^{\circ} = \rho_3^{\circ}$$

$$0 \le \sigma^{\circ} \le \pi$$

81

DELETED...

ANGLE BETWEEN TABLE ρ_1 AXIS AND PROJECTION OF SUN ON PLANE NORMAL TO LEM LINE-OF-SIGHT

83

$$\tan\gamma^{\circ} = \frac{\rho_2^{\circ}}{\rho_1^{\circ}}$$

$$0 \le \gamma^{\circ} < 2\pi$$

SUN'S DIRECTION MEASURED IN RENDEZVOUS (TABLE-COORDINATES) FRAME IN P

84

$$\rho_1^{\circ} = P_{11} X^{\circ}_{B/C} + P_{21} Y^{\circ}_{B/C} + P_{31} Z^{\circ}_{B/C}$$

$$\rho_2^{\circ} = P_{12} X^{\circ}_{B/C} + P_{22} Y^{\circ}_{B/C} + P_{32} Z^{\circ}_{B/C}$$

$$\rho_3^{\circ} = P_{13} X^{\circ}_{B/C} + P_{23} Y^{\circ}_{B/C} + P_{33} Z^{\circ}_{B/C}$$

P^{T}_{ij} (J-61)

SUN'S DIRECTION MEASURED IN CSM BODY-B FRAME

85

$$\begin{pmatrix} X^{\circ}_{B/C} \\ Y^{\circ}_{B/C} \\ Z^{\circ}_{B/C} \end{pmatrix} = \left[g_{ij_c} \right] \begin{pmatrix} X_{n/0}/r_{n/0} \\ Y_{n/0}/r_{n/0} \\ Z_{n/0}/r_{n/0} \end{pmatrix}$$

$(X,Y,Z,r)_{n/0}$ (J-31, E-30)

$[g_{ij_c}]$ (J-62)

ANGLE BETWEEN $F_{EN}, F_{M/V}$ AND SUN'S DIRECTION FROM EARTH, MOON

87

$$\sigma_n^{\circ} = \cos^{-1}\left[\frac{X_{n/0}X_{n/V} + Y_{n/0}Y_{n/V} + Z_{n/0}Z_{n/V}}{r_{n/V}\, r_{n/0}}\right], \quad 0 \le \sigma_n^{\circ} \le \pi$$

$$R^{*}_{n/0} = r_{n/V} \sin\sigma_n^{\circ} \qquad V = L \text{ or } C$$

NOTE: OCCULTATION DATA FOR THE LEM MAY BE DETERMINED FROM J-86 AND J-87,88 BY REFERENCING POSITION PARAMETERS WITH RESPECT TO SUBSCRIPT L (LEM) RATHER THAN SUBSCRIPT C (CSM) (FOR POSSIBLE USE WITH J-30).

$(X,Y,Z,r)_{M/0}$ (J-31)

$(X,Y,Z,r)_{E/0}$ (E-30)

$(X,Y,Z,r)_{n/V}$ (A-12,13)

L&A SHADOW OF LEM ON SURFACE

POSITION OF SHADOW IN BODY FRAME

91

$$\begin{pmatrix} P_{XLA} \\ P_{YLA} \\ P_{ZLA} \end{pmatrix} = \left(V^{*}_{gk}\right) \begin{pmatrix} X_{LAS} \\ Y_{LAS} \\ Z_{LAS} \end{pmatrix}$$

V^{*}_{gk} (J-47)

POSITION OF SHADOW IN TERRAIN COORDINATES

90

$$X_{LAS} = h_{LAK}$$

$$Y_{LAS} = \frac{X_{LAS}\cos\mu^{\circ}}{\tan\theta^{\circ}}$$

$$Z_{LAS} = \frac{X_{LAS}\sin\mu^{\circ}}{\tan\theta^{\circ}}$$

h_{LAK} (G-30)

$\mu^{\circ}, \theta^{\circ}$ (J-01)

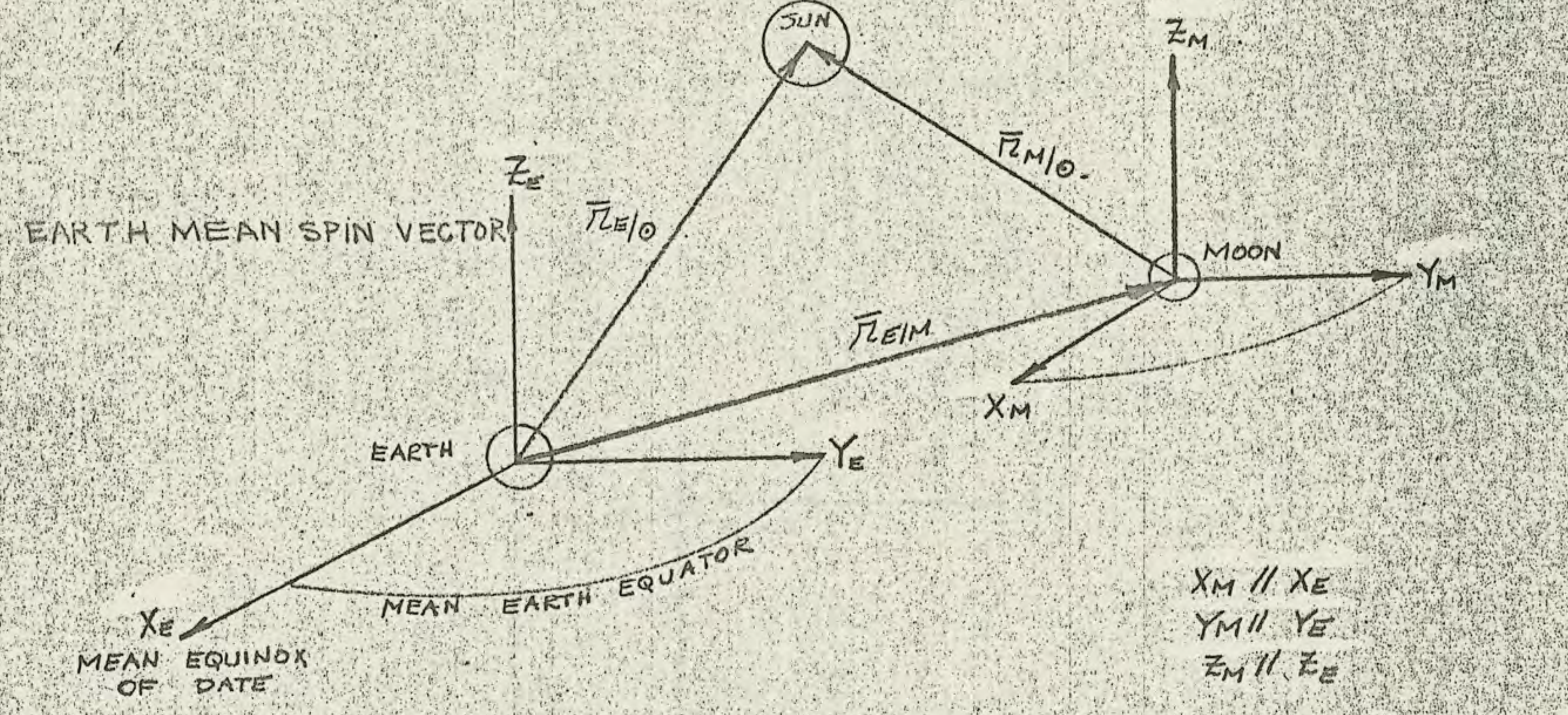


FIGURE-1. INERTIAL EARTH E AND MOON M-FRAME GEOMETRY

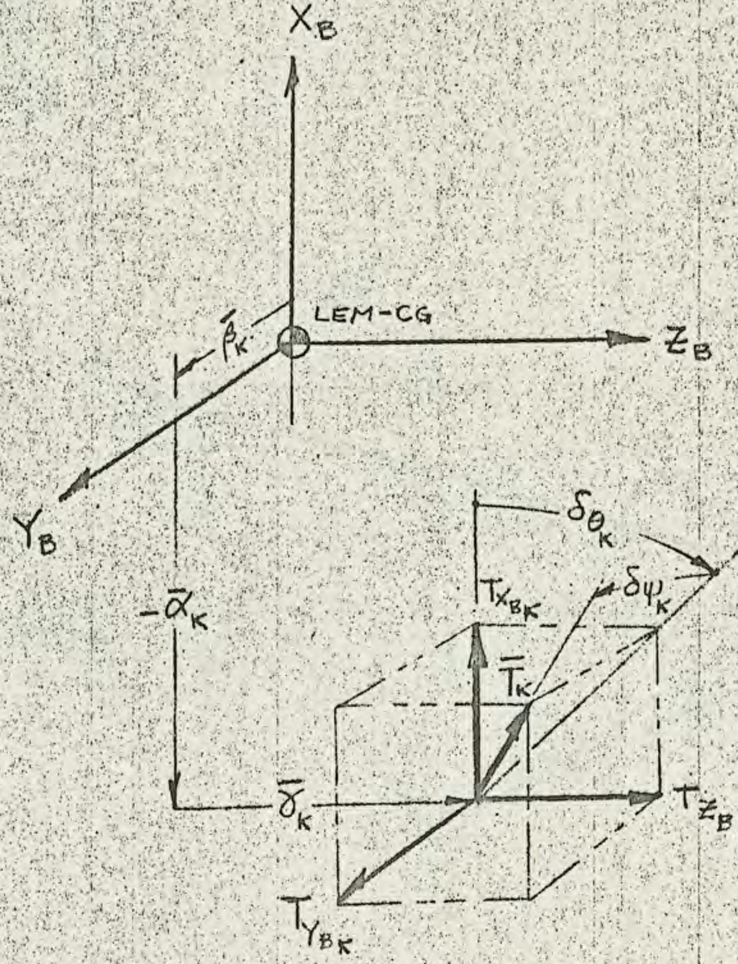
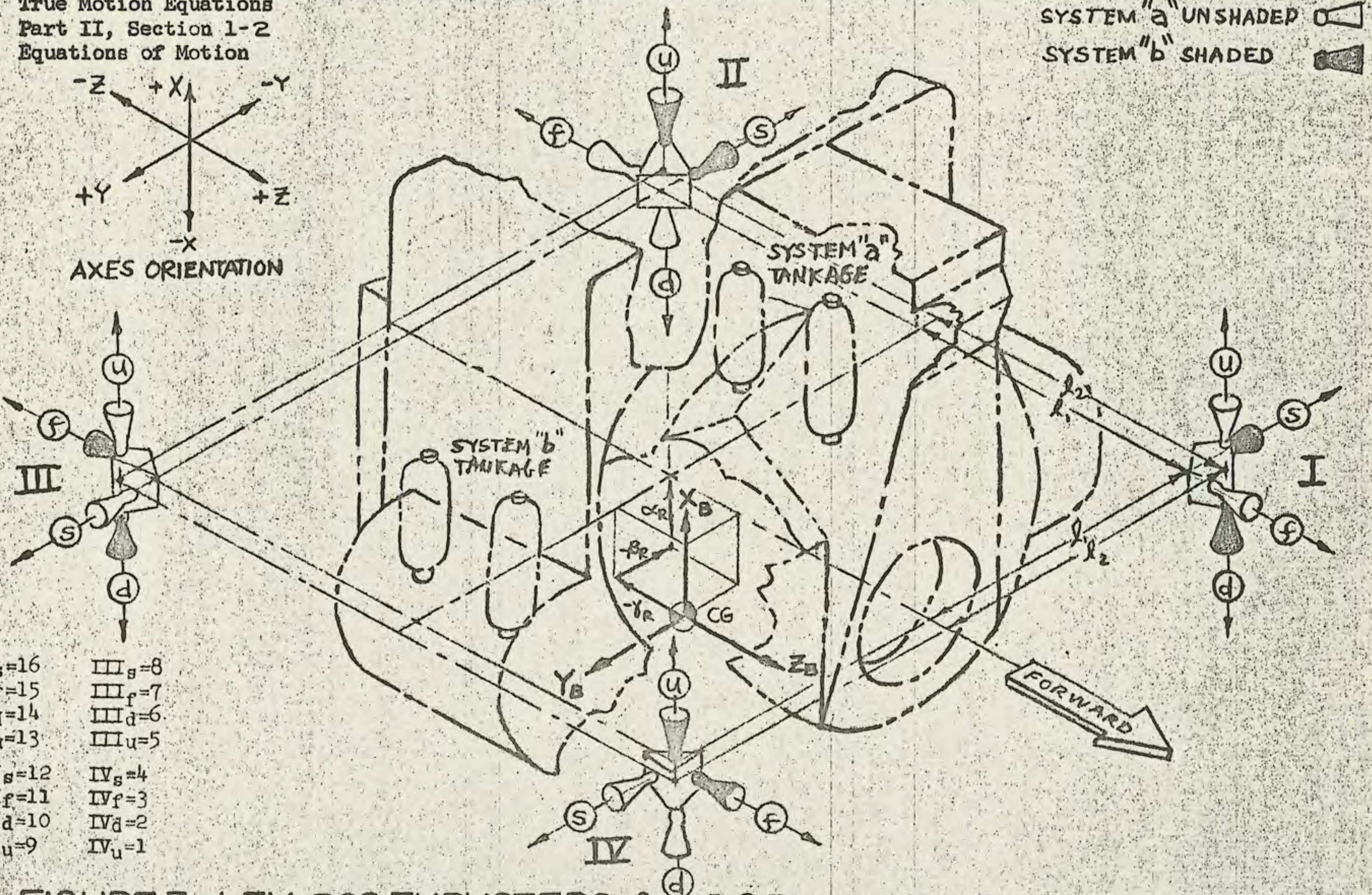


FIGURE-2. MAIN ENGINE THRUST GEOMETRY

True Motion Equations
Part II, Section 1-2
Equations of Motion

SYSTEM "a" UNSHADED
SYSTEM "b" SHADED



$I_s=16$
$I_f=15$
$I_d=14$
$I_u=13$

$II_s=12$
$II_f=11$
$II_d=10$
$II_u=9$

$III_s=8$
$III_f=7$
$III_d=6$
$III_u=5$

$IV_s=4$
$IV_f=3$
$IV_d=2$
$IV_u=1$

FIGURE-3. LEM RCS THRUSTERS & BODY B-FRAME GEOMETRY

Figure 4. Not used.

c

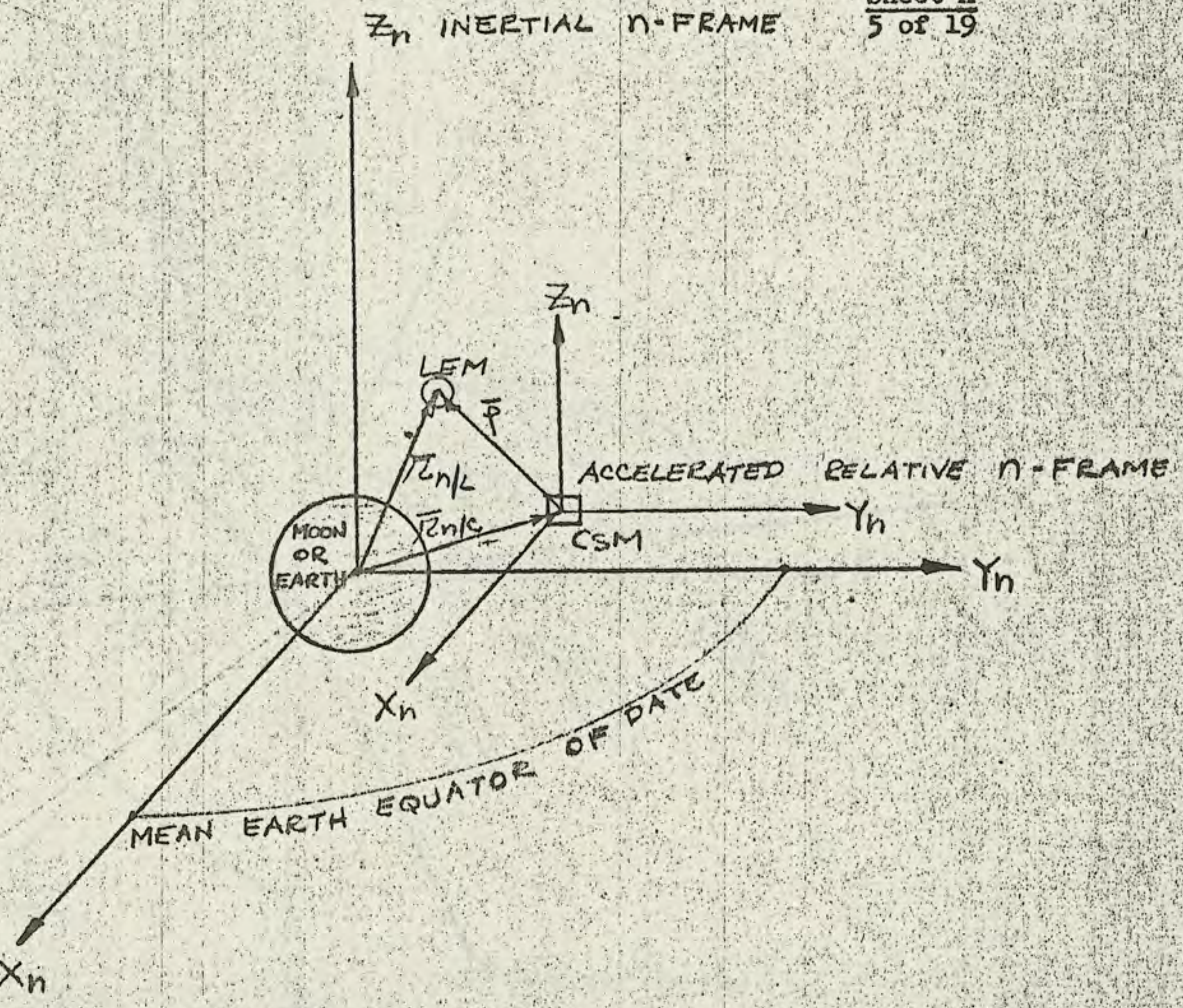


FIGURE-5. INERTIAL AND RELATIVE M OR E-FRAME SCHEMATIC

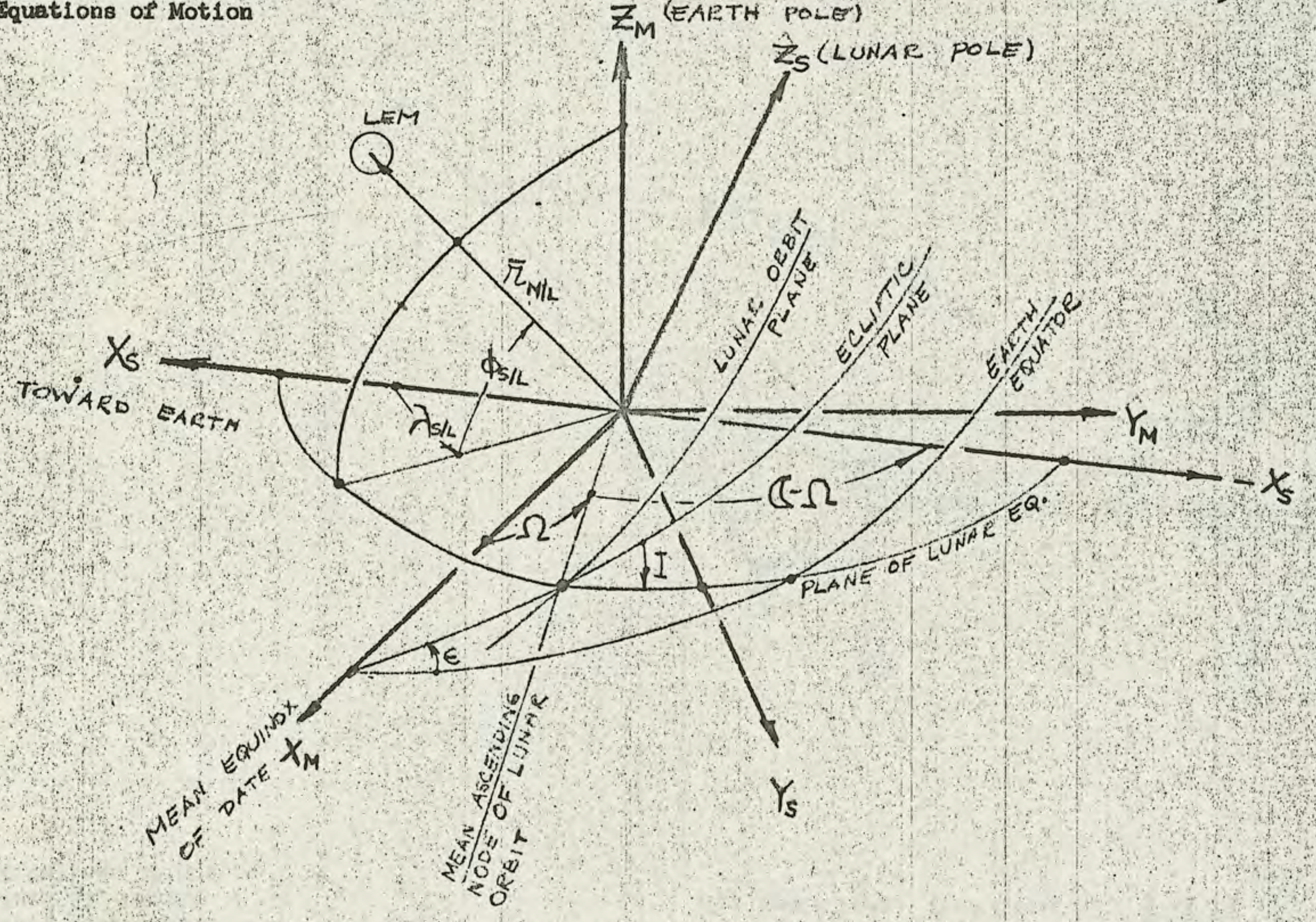
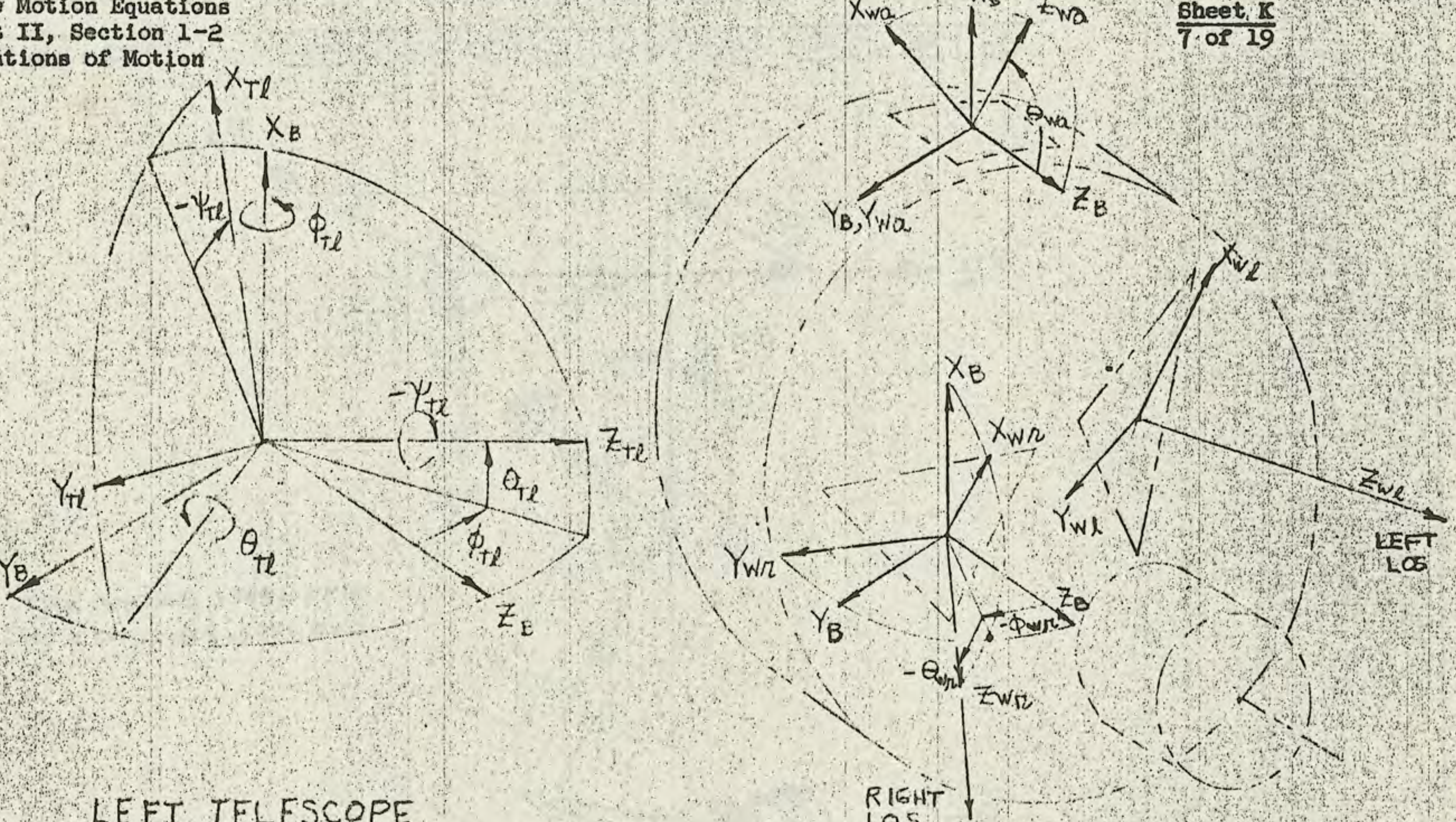


FIGURE-6. RELATION BETWEEN M-FRAME AND SELENOGRAPHIC S-FRAME

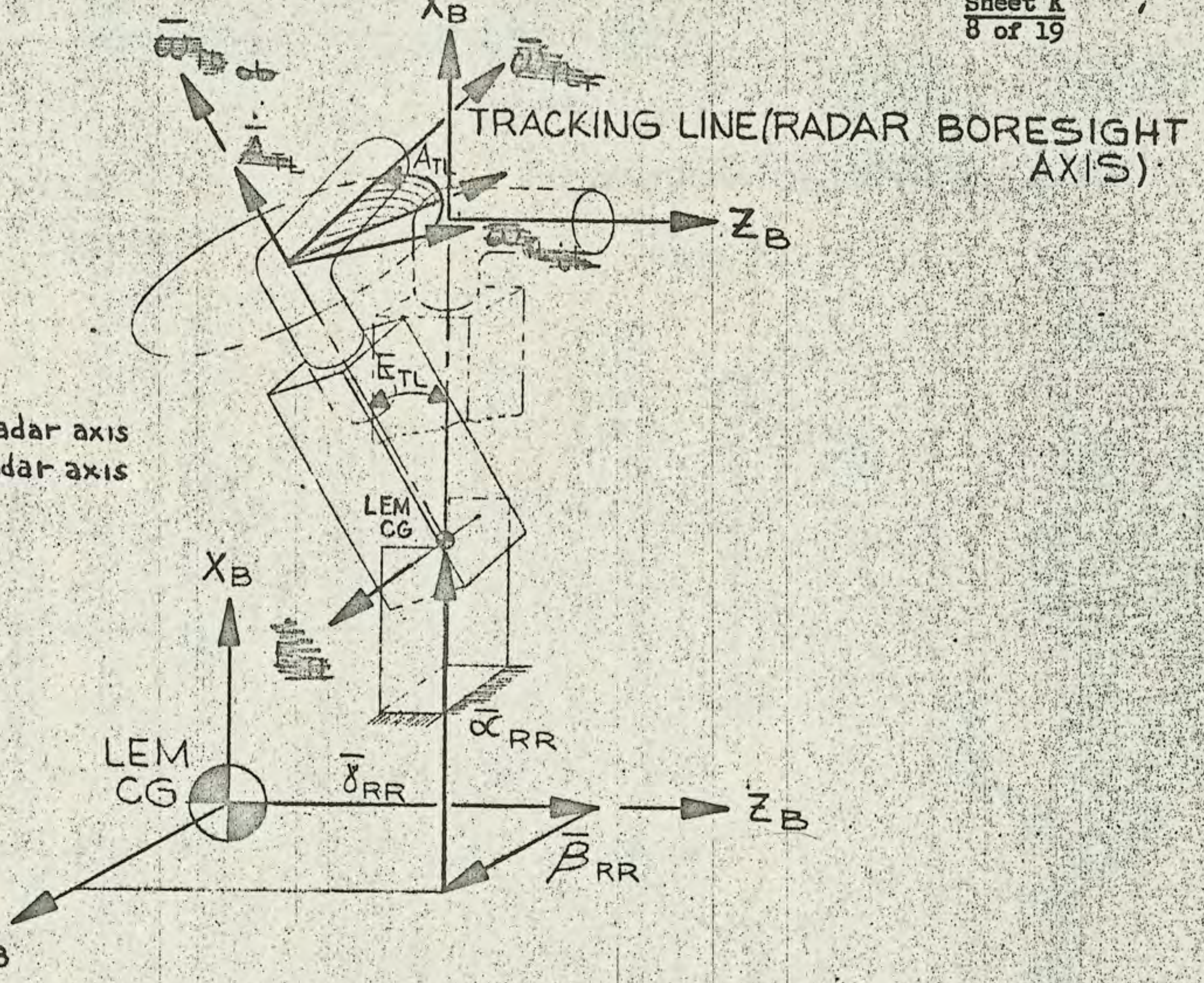
1681



LEFT TELESCOPE OPTICAL AXES

WINDOW OPTICAL AXES

FIGURE -7. LEM OPTICAL AXES



ob= outboard radar axis
ib= inboard radar axis

FIGURE-8. RENDEZVOUS RADAR GEOMETRY

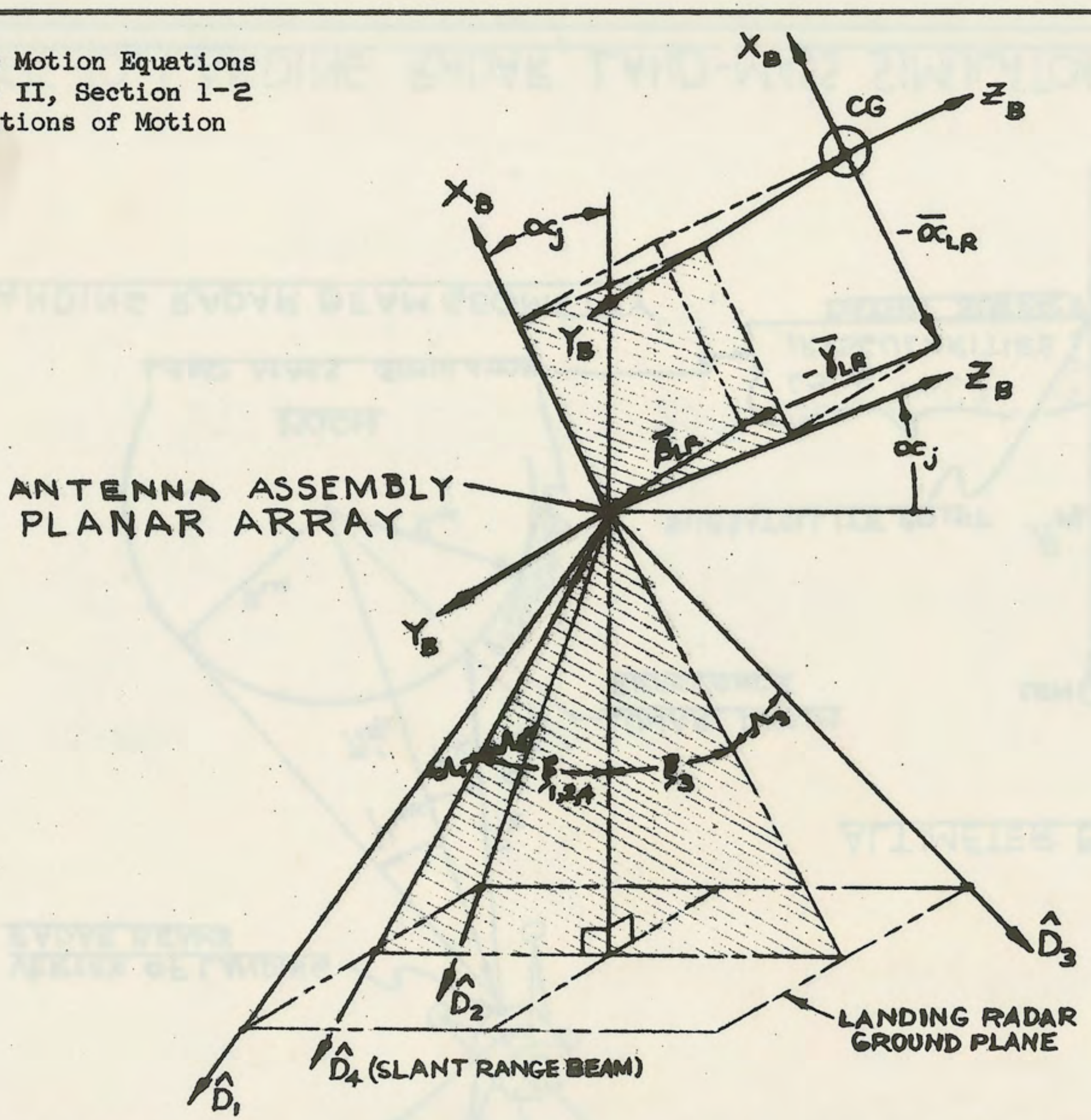


FIGURE-9. LEM LANDING RADAR GEOMETRY

1122

6

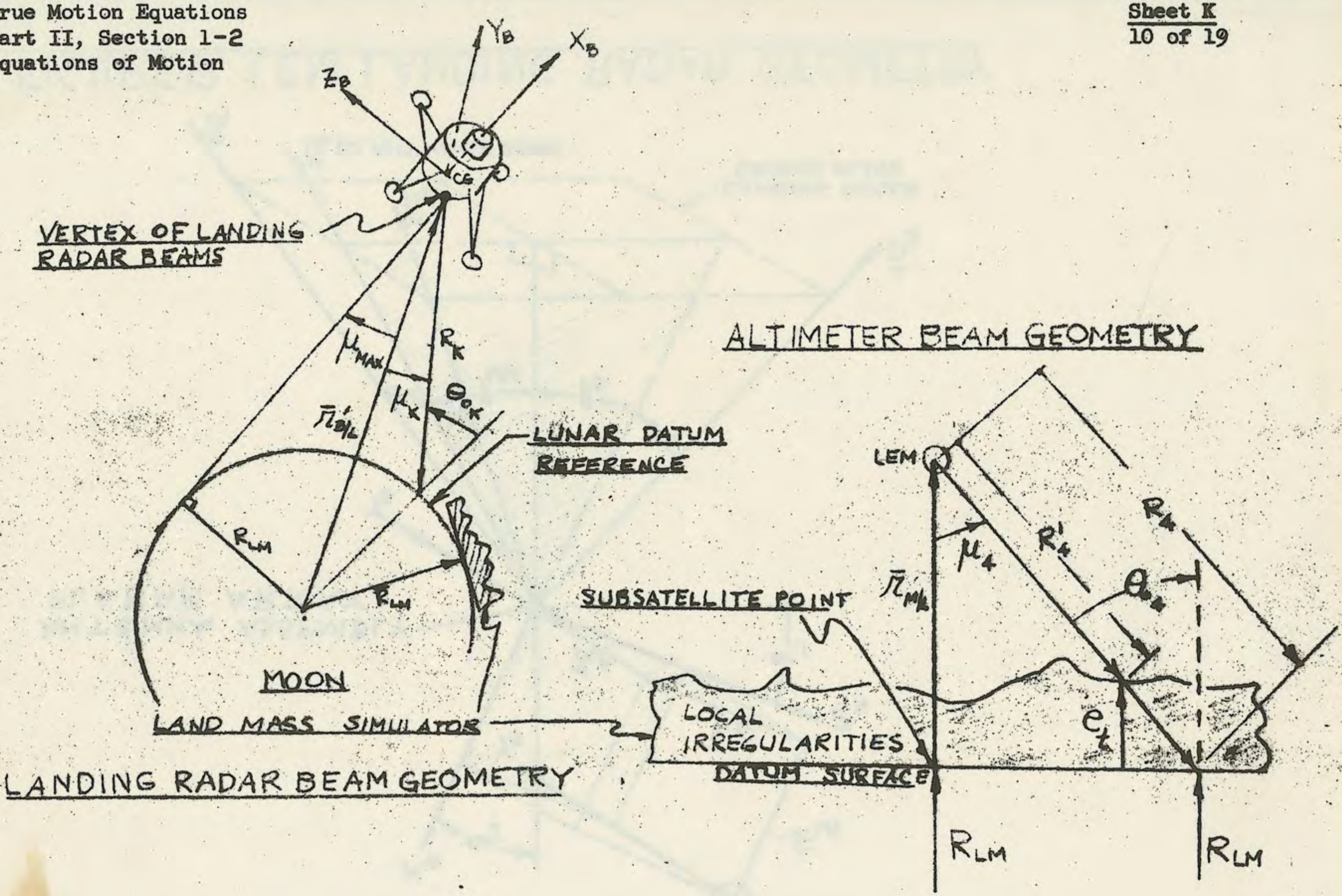


FIGURE-10. LANDING RADAR, LAND-MASS SIMULATOR (BEAM GEOMETRY)

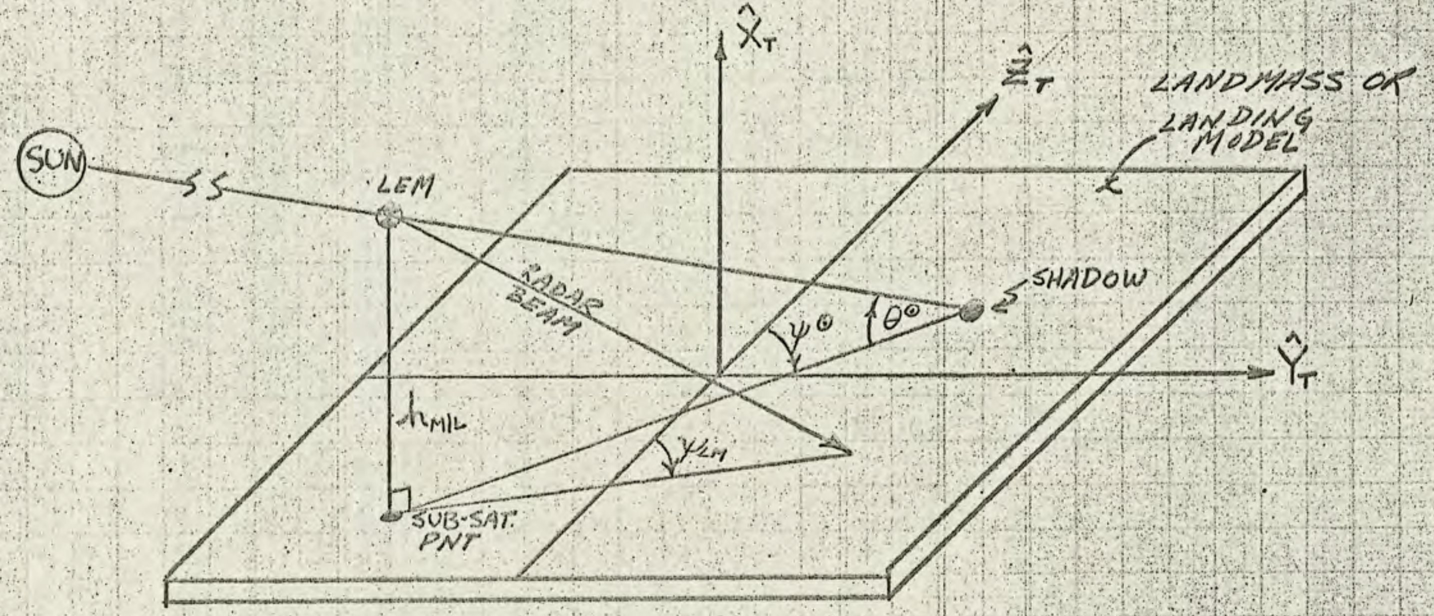


FIGURE - 11. EVDE SHADOW GENERATION AND LAND MASS SIMULATOR BEAM DIRECTION

Figure 12. Not used.

1122c

1123

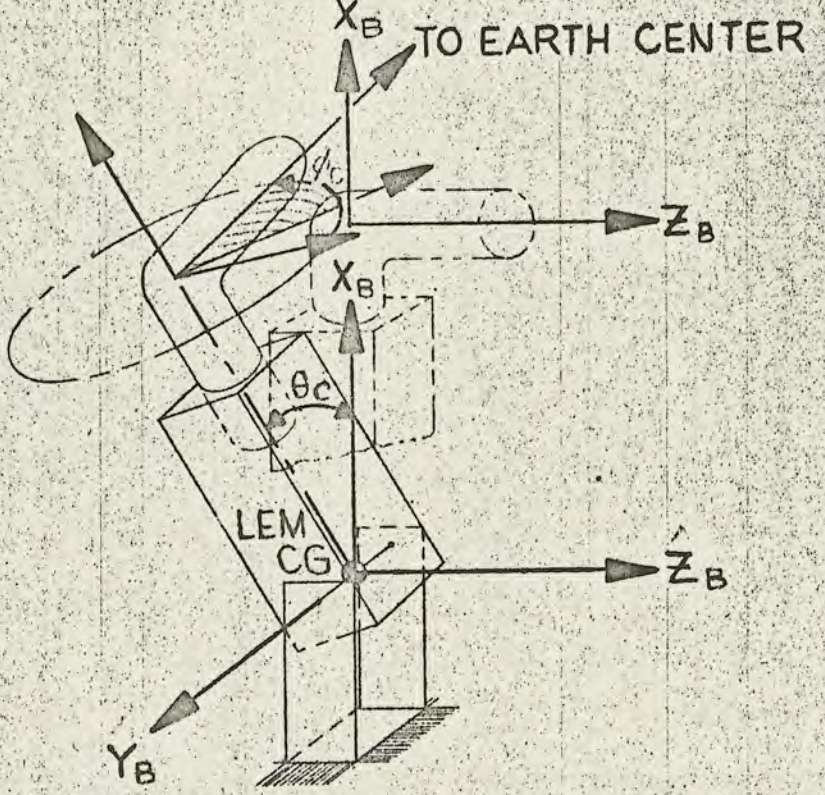
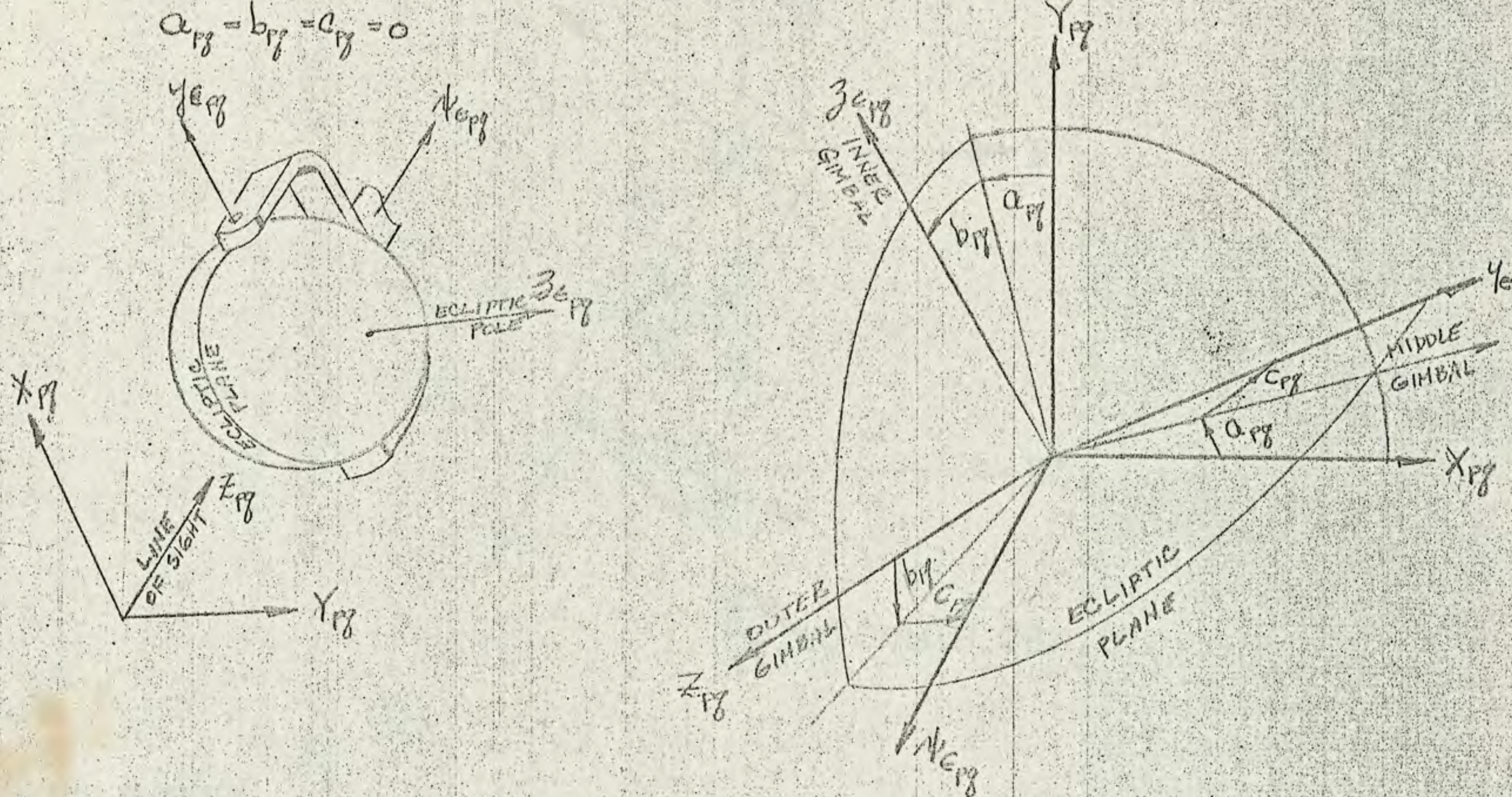


FIGURE-13. EARTH COMMUNICATION RADAR GEOMETRY



a. CELESTIAL SPHERE ASSEMBLY

b. GIMBAL ROTATIONS

FIGURE-14 CELESTIAL SPHERE GIMBAL DRIVES

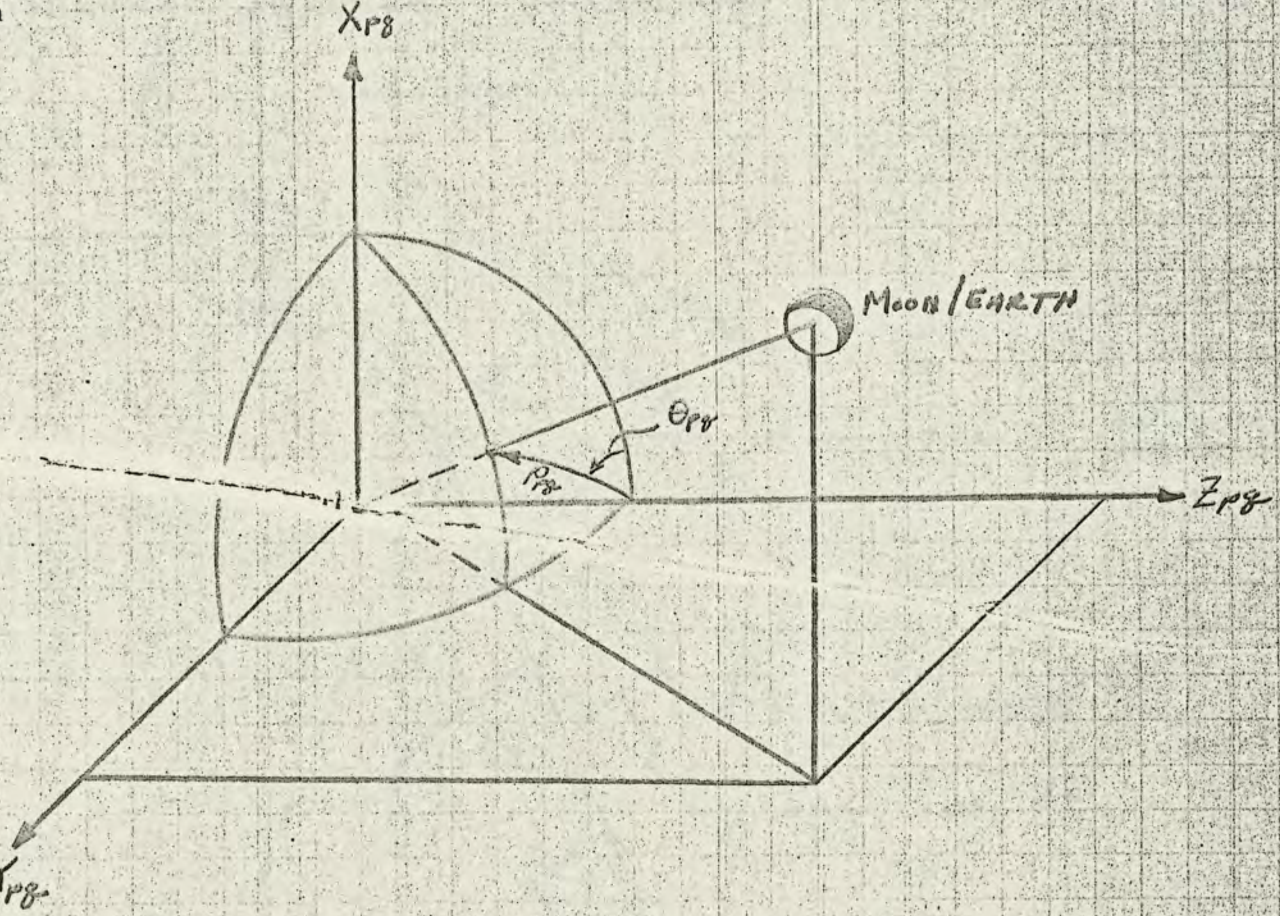


Figure-15. OCCULTER DRIVES