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Subject: EFFECT OF LMS FIFTY-MILLISECOND INTEGRATION STEPS ON SIMULATED RESPONSE OF ABORT ATTITUDE CONTROL SYSTEM

References:

(1) LAV-440-43, A. L. Reynolds/T. Woods, "Study of Effect of Computation Rates on Simulated LEM Response for the LMS", October 27, 1964.

(2) LMO-500-122, R. Portnoy/H. Satz, "LEM, CSM, and SIVB Attitude Control System Digital Subroutines", January 27, 1964.

(3) LMO-500-246, R. Bergeman, "Comparison Studies of Descent Engine Trim System Duty Factors and Switching Requirements for Alternate Stabilization Techniques", February 11, 1965.

Summary:

The problem of accurately simulating the analog LEM abort attitude control system in the "real time" LEM Mission Simulator (LMS) digital computer which uses a 50 millisecond integration step has been investigated. The analysis was constrained to finding a technique which minimizes the ratio of the calculation time of all necessary equations to the iteration interval (50 ms). This number must obviously be less than 1.0 for a real time simulation.

This memo presents a technique for modeling the LEM abort attitude control system consistent with the operational constraints imposed by the LMS design concept. It was found that a Pulse Ratio Modulator (PRM) on-time integration factor which is an experimentally-derived function of vehicle inertia and moment unbalance (derived from center-of-gravity offset) must be introduced for the ascent phase of the mission to help compensate for the discrete calculations at the large step size (50 ms.) of the analog attitude control system. No integration factor is required for the descent phase. Sample curves for this factor are included. This integration factor somewhat restricts the simulation's versatility in that curves for the correction factor as a function of moment unbalance and vehicle inertia must be rederived each time any of the system parameters are changed.

The included program can be easily extended to the 3-axis case, but further study with the inclusion of jet select logic will be necessary to demonstrate its accuracy. In addition, angular accelerations are held constant over each integration period (as is the case in the present program), permitting integration procedures in the 3-axis case to remain relatively simple.

Introduction:

The LEM Mission Simulator (LMS), which will be used in astronaut training, is being constructed by Link from a math model supplied by GAEC. The simulator will employ digital computers to process all vehicle state equations, including those representing the analog abort control system.

At present, a basic integration step of 0.05 sec. is being considered for computation of all the simulation equations, including those representing vehicle motion and the abort attitude control system. That is, e.g., once every 50 ms. pilot commands, the vehicle state vector, and mode of operation information are fed into the attitude control segment of the simulator and combined with the other outputs (propulsion, slosh, etc.), and updated vehicle angular rates and positions and RCS propellant consumption information are released.

A study to determine the simplest set of programmable equations that accurately represents the abort attitude control system performance (in terms of vehicle motion and RCS propellant consumption) was undertaken in response to a study request (Reference 1) by LEM Training Equipment. By "simplest set" it is meant the set of equations that can be solved in the shortest time, thereby minimizing the ratio of equation solution time to integration step size. The equations in the high iteration rate program (1000 times per sec. necessary to accurately represent the analog system - see Reference 2) which has been used in the past to examine the abort control system would have to be recalculated many times each 50 ms., probably causing the ratio of total calculation time to the 50 ms. integration step to be too large to make their use practical. Also, this aforementioned program's integration steps cannot be increased to anywhere near 50 ms. without causing gross inaccuracies in vehicle performance when compared to the "real world". A model suitable for digital programming which compensates for these inaccuracies and can operate in a real time simulation must be constructed. This memo describes various methods that were used to simplify the system for simulation purposes, interpolate within the 50 ms. integration steps, and compensate for inaccuracies. A digital program which incorporates the recommended modeling scheme is outlined in detail in the Appendix.

The study was restricted to a single axis LEM motion with cross-coupling terms, bending, and slosh dynamics neglected. Command signals, vehicle inertia, center-of-gravity positions, and main engine thrust were held constant in each computer run.

Discussion:

A block diagram of the LEM abort attitude control system for single-axis rotation is presented in Figures 1 and 2. Reference 2 contains a detailed description of the elements outside the Gimbal Trim System and Reference 3 describes the GTS. Figure 3 illustrates the delays between the PRM output and the RCS jets' thrust.

In the real world, the entire system is continuous (except for the attitude feedback, which is sampled 25 times/sec. - near the iteration rate of the LMS). PRM on and off times are determined by the analog integration of equations shown in Figure 2. Each time an on-or off-time integral is fulfilled, the PRM output changes state and integration of the other equation is begun.

In a digital simulation, the differential equations representing the time histories of the various signals in the system must be replaced by difference equations which are recalculated at the desired iteration rate. The digital program that has been used in the past to simulate the abort control system employed an integration step of 0.001 sec. Various techniques of altering this program and simplifying the system to permit the use of 50 ms. integration steps (and still realize an accurate system) were investigated in the present study and are outlined below, along with their drawbacks. Technique No. 4 is the one recommended. (The inherent slow response time of the GTS permits the equations describing the GTS in the high-accuracy program to be iterated at much lower rates with no loss in accuracy, so no simplification techniques are needed for the GTS).

1. The integration step in the high-accuracy simulation of Reference 2 was increased from 1 ms. to an integral submultiple of 50 ms. and the relevant variables were printed out every 50 ms.

   The integration step can be increased to about 5 ms. without any deterioration in accuracy, but this still requires going through the governing set of equations 10 times every 50 ms., involving about 600 x 10 = 6000 computer operations, when FORTRAN II is used.

2. RCS filter and rate gyro dynamics and the RCS jets output delays were omitted, making the PRM input a simple function of vehicle angular position and rate. This input was inserted into the PRM equations, which were integrated to find the times at which jets were turned on or off. The equations of motion were integrated each time this happened, and every 50 ms. in between.

   The set of equations that must be recalculated every 50 ms. is fairly short if the jets remain either on or off during the calculation period, but becomes very involved if the jets' outputs change state during this time, cube root calculations being necessary. To compensate for the above-mentioned omissions, it was also necessary to make the integration factor (λ) in the jet off-time integral an experimentally-derived function of vehicle inertia and moment unbalance, the two governing independent variables.

3. Again omitting filter and gyro dynamics and system delays, jet off-times were roughly calculated to the nearest multiple of 50 ms. - using an estimated average PRM input signal during each 50 ms. as the integrand in the off-time equation. Jet on-time integration was omitted completely and thrusting periods were confined to either 50 or 100 ms. The thrust amplitude was made a function of the PRM input level at the beginning of each firing, and experimentally-derived off- and on-time integration factors were injected to compensate for all inaccuracies.

The set of equations to be gone through each 50 ms. is very small, but the integration factors are discontinuous functions of inertia and moment unbalance, and interpolation is difficult. Also, accuracy during transient responses is poor.

4. RCS filter and gyro dynamics were included to permit accurate (when compared to the "real world") computation of the PRM input. An average input for each 50 ms. period was predicted by straight-line interpolation between known beginning and computed end values and was integrated to find when the on-or off-time integrals were exceeded. When this occurred, interpolation was used to find the exact time when the integral was fulfilled (straight line interpolation avoids the complication of taking cube roots), the appropriate RCS jet delay was added in and these times were taken into account in the equations of motion. Vehicle angular acceleration was held constant over the entire integration interval.

   The inaccuracies caused by straight-line interpolation between PRM input values must sometimes be offset by inserting an experimentally-derived correction parameter. The most logical choice is an on-time integration factor, the largest inaccuracies occurring during these periods. This factor is plotted as a function of moment-unbalance - caused acceleration at sample ascent inertias in Figure 2A (it is also an implicit function of all other present nominal system parameters). The curves are continuous and with the addition of more points, interpolation would not be too difficult. No correction factors are needed for the descent phase, with or without the GTS. The set of equations to be recalculated every 50 ms. can be made fairly small; about 1,250 computer operations are involved, if FORTRAN II is used and no efforts are made to determine the integration factor analytically.

Results

Simulation Method No. 4 (see Discussion) was used in runs covering the ascent and descent ranges of inertias, in the rate command and attitude hold portions of the Attitude Hold Mode of operation. Correspondence of angular rate and displacement time histories between these runs and those from the high-accuracy program was excellent for ascent inertias (when the on-time integration factor was included), noticeable differences occurring only in the initial transient response of each run. Propellant consumption errors were negligible.

The omission of an on-time integration factor in runs involving descent inertias (with and without the GTS) produced slight errors in rate and displacement time histories, and propellant consumption figures were in error by a maximum of about 7.5 percent. These errors were considered small enough to warrant the integration factor's omission.

LIST OF SYMBOLS, UNITS, AND PRESENT VALUES

Symbol	Definition	Units and Present Nominal Value (if applicable)
C	System Inputs	Deg. - Attitude Hold Mode; Deg./Sec. - Rate Command Mode
DDLTAC	System Command Level	0.2 Deg.
DLTLIM	Gimbal Trim System Output Limit	6 Deg.
F	RCS Jets' Thrust (2 Jets)	200 Pounds
GDZ	Gimbal Deadzone	0.375 Deg.
GL	Distance from Gimbal Pivot Point to X-axis Center of Gravity	5 Ft. Max., 2 Ft. Min. Descent
GTC	Gimbal Time Constant	0.1 Sec.
GTS	Gimbal Trim System	
L	RCS Jet Lever Arm	5.5 Ft.
PRM	Pulse Ratio Modulator	
QS	Rate Gyro Saturation Point	20 Deg.
RCS	Reaction Control System	
S	Differential Operator	
SMOM	Moment Unbalance = BIGF*Ø	Ft.-lbs.
T_1	RCS Jet Turn-on Delay	0.009 Sec.
T_2	RCS Jet Turn-off Delay	0.005 Sec.
TW	PRM Output Pulse Length	Sec.
TWMIN	Minimum PRM Output Pulse Length	0.01 Sec.
w_f	RCS Filter Undamped Frequency	113 Rad./Sec.
w_g	Rate Gyro Undamped Frequency	125 Rad./Sec.
z	Damping Ratio	0.8
θ	Vehicle Attitude	Deg.
$\dot{\theta}$	Angular Rate = $d\theta/dt$	Deg./Sec.
λ	PRM Nonlinearity Factor	0.1
ϕ	Y- or Z-axis Center-of-Gravity Offset	Feet

LIST OF SYMBOLS, UNITS, AND PRESENT VALUES (Cont'd)

Symbol	Definition	Units and Present Nominal Value: Descent	Units and Present Nominal Value: Ascent	
BIGF	Main Engine Thrust	10,500 Max. 1,050 Min. (Powered)	3500	Pounds
DZ	PRM Input Deadzone	0.3 (Powered) 5.0 (Coasting)	0.3 (Powered) 5.0 (Coasting)	Deg.
I	Vehicle Inertia about any Major Axis	24,780 Max. 11,790 Min.	6,174 Max. 1,530 Min.	Slug-ft^2
LIM	Angular Error Limit	15	4	Degrees
RG	Rate Gyro Gain	1.5	0.4	
SAT	PRM Input Saturation Point	0.8 (Powered) 5.5 (Coasting)	0.8 (Powered) 5.5 (Coasting)	Deg.

APPENDIX

RECOMMENDED METHOD OF SOLUTION

Introduction

The logic and equations that are recommended for use in digitally simulating the analog abort attitude control system (with constant inertia, center-of-gravity positions, and input command) when using the LMS 50 ms. integration steps are presented herein. The equations represent a single axis case only, so that 3 iterations, plus the jet select logic, will be necessary to provide 3-axis information. In that angular acceleration is held constant over each integration interval, the complete 3-axis system should perform well. A simplified digital program flow chart is presented in Figure 1A. Blocks in the flow chart which involve detailed sets of equations have encircled letters beside them. Each letter is called out in one of the sections following this introduction, and the corresponding set of equations is discussed in that section. Figure 2A shows the experimentally-derived on-time integration factor (B) plotted as a function of moment unbalance at sample ascent inertias. No correction factors are necessary for the descent case.

The following symbols are used exclusively in the Appendix and are here defined:

Symbol	Definition
a	Position of dummy switch in flow chart (see Figure 1A) when GTS is not being used
b	Position of switch when GTS is in use
B	PRM on-time integration factor (see Figure 2A for sample curves)
BETA	(F * L/I) * 57.29578
d	GTS motor output
FUEL	Remaining propellant
M	Dummy variable which is set equal to 1 to turn the jets on and is set equal to zero to turn the jets off.
N	Dummy variable which is set equal to 1 just before propellant consumption calculations are to be made (at the end of each thrusting period) and is set equal to zero at all other times
T	Present time
T_K	Time at which jets turn on
T_O	Time one integration interval behind T (except for initial conditions, where $T = T_O = 0$.)
q	Rate gyro output
y	RCS filter output
α	(SMOM/I) * 57.29578
ΔT	$T - T_O$ (50 milliseconds, except at $T = 0$.)

Symbol	Definition
Δ TW	Incremental addition to an on or off-time PRM integral
$\Delta\tau$	Interval used in integrating equations for q, $\dot{q}$, y, $\dot{y}$, θ, $\dot{\theta}$ (see Section B for further explanation)
$\ddot{\theta}$	Total angular acceleration (in deg./sec^2.) - constant during any interval under consideration

FLOW CHART BLOCK A (FIGURE 1A) - INITIAL CONDITIONS AND DEFINITIONS

The following procedures apply only to the present single axis program, and need not be followed in the order presented here. In a complete simulation, many of these inputs will come from other sections of the LMS.

Read in the control system parameter values that are constant for all phases of the mission (listed at the beginning of this memo). Choose a mission phase (powered or coasting ascent, or powered or coasting descent), set in the corresponding values of DZ, SAT, RG and LIM, and select the attitude hold or rate command mode of operation. (If powered descent is being investigated, set in the desired BIGF and GL+) Choose an operating I and a ∅+ (if a powered phase of the mission is being examined) and obtain the correct integration factor (B) from Figure 2A. Choose an initial angular position and rate, initial filter, GTS, and rate gyro outputs, values for M and N, and an initial amount of propellant. Set T and T_o equal to zero.

The following variables are defined for use in later calculations:

$$a_1 = -z * w_g + w_g * \sqrt{(z^2 - 1.)}$$

$$b_1 = -z * w_g - w_g * \sqrt{(z^2 - 1.)}$$

$$a_2 = -z * w_f + w_f * \sqrt{(z^2 - 1.)}$$

$$b_2 = -z * w_f - w_f * \sqrt{(z^2 - 1.)}$$

FLOW CHART BLOCK B (FIGURE 1A) - EQUATIONS FOR RCS FILTER OUTPUT

1. Rate Feedback Loop (see Figure 1)

The rate gyro output limit (QS) is rarely reached in nominal operation of the control system, so this limit is omitted to facilitate computations. With this omission, the rate feedback loop transfer function becomes:

$$\frac{q}{\dot{\theta}} = \frac{RG * w_g^2}{S^2 + 2 * z * w_g * S + w_g^2} \qquad q = \text{rate gyro output}$$

Defining
t = time
$\dot{\theta}(0)$ = angular rate at t = 0.
q(0) = gyro output at t = 0.
$\dot{q}(0)$ = derivative of gyro output at t = 0.

+ In a complete simulation, I, GL, and ∅ will have to be continuously recomputed.

$$c2 = \left(\frac{a_1}{a_1 - b_1}\right) * \left(- RG*\dot{\theta}(0) + q(0) - \dot{q}(0)/a_1 + RG*\ddot{\theta}* \left\{\frac{1}{a_1} + \frac{2*z}{w_g}\right\}\right)$$

$c_1 = \left(- \frac{1}{a_1}\right) * (b_1 * c_2 + RG * \ddot{\theta} - \dot{q}(0))$, the equation for q and its derivative as a function of time is

$$q(t) = c_1 * e^{a_1 t} + c_2 * e^{b_1 t} + RG * \dot{\theta}(0) - (2 * z * RG * \ddot{\theta}/w_g) + RG * \ddot{\theta} * t$$

and

$$\dot{q}(t) = a_1 * c_1 * e^{a_1 t} + b_1 * c_2 * e^{b_1 t} + RG * \ddot{\theta}$$

($\ddot{\theta}$ is assumed to remain constant during the time under consideration.)

The following section will describe the programmable form of these equations.

2. <u>RCS Filter Section</u> (see Figure 1)

The angular error limit (LIM) is reached only in the initial portions of a transient response following a large input, and is therefore omitted to facilitate computations. With this omission the filter transfer function becomes:

$$\frac{y}{E} = \frac{w_f^2}{S^2 + 2 * z * w_f * S + w_f^2}$$

E = filter input = $-q - \theta$ (C = 0.)

y = filter output

Defining

$y(0)$ = filter output at t = 0.

$\dot{y}(0)$ = derivative of filter output at t = 0.

$B_2 = -RG * \dot{\theta}(0) + (2 * z * RG * \ddot{\theta}/w_g) - \theta(0)$ for attitude hold mode

$B_2 = -RG * \dot{\theta}(0) + (2 * z * RG * \ddot{\theta}/w_g)$ for rate command mode

$B_3 = -RG * \ddot{\theta} - \dot{\theta}(0)$ for attitude hold mode

$B_3 = -RG * \ddot{\theta}$ for rate command mode

$B_4 = -\ddot{\theta}/2$ for attitude hold mode

$B_4 = 0.$ for rate command mode

$$M_1 = B_2 - (2 * z * B_3/w_f) + (2 * B_4 * \{4 * z^2 - 1.\} /w_f^2)$$

$$M_2 = B_3 - 4 * z * B_4/w_f$$

$$M_3 = B_4$$

$$M_4 = -(w_f^2 * c_1)/(w_f^2 + 2 * z * w_f * a_1 + a_1^2)$$

$$M_5 = -(w_f^2 * c_2)/(w_f^2 + 2 * z * w_f * b_1 + b_1^2)$$

$$D_2 = \left(\frac{1}{b_2 - a_2}\right) * (\dot{y}(0) - M_2 - a_1 * M_4 - b_1 * M_5 + a_2 * \{M_1 + M_4 + M_5 - y(0)\})$$

$D_1 = -D_2 - M_1 - M_4 - M_5 + y(0)$, the equation for y as a function of time is:

$$y(t) = D_1 * e^{a_2 t} + D_2 * e^{b_2 t} + M_1 + M_2 * t + M_3 * t^2 + M_4 * e^{a_1 t} + M_5 * e^{b_1 t}$$

$$\dot{y}(t) = a_2 * D_1 * e^{a_2 t} + b_2 * D_2 * e^{b_2 t} + M_2 + 2 * M_3 * t + a_1 * M_4 * e^{a_1 t} + b_1 * M_5 * e^{b_1 t}$$

($\ddot{\theta}$ is assumed to remain constant during the time under consideration)

Now, if t is defined as being zero at T_o, and y is to be calculated at a time $\Delta\tau$ later, then $q(0) \longrightarrow q(T_o)$, $y(0) \longrightarrow y(T_o)$, etc. and

$$y(T_o + \Delta\tau) = D_1 * e^{a_2 \Delta\tau} + D_2 * e^{b_2 \Delta\tau} + M_1 + M_2 * \Delta\tau + M_3 * (\Delta\tau)^2 + M_4 * e^{a_1 \Delta\tau} + M_5 * e^{b_1 \Delta\tau}$$

Equations for $\dot{y}$, q, $\dot{q}$, θ, and $\dot{\theta}$ can be represented the same way, and will then be in proper form for use in the digital program.

In order to simplify all integrations, $\ddot{\theta}$ is held constant during each 50 ms. interval. Therefore, when it is discovered that a change in θ should take place during an interval, this event is suppressed until the end of the interval. In order to prevent this suppression from causing errors, the integration step ($\Delta\tau$) used in the equations for q, $\dot{q}$, y, $\dot{y}$, θ, and $\dot{\theta}$ is varied, even though these equations are recomputed only at 50 ms. intervals.

When it is calculated that the RCS jets should begin firing at a time T_{fire} ($T_o < T_{fire} < T$), they are suppressed until time = T, an integration step of $\Delta\tau = T_{fire} - T_o$ is used in the equations for q, $\dot{q}$, etc., and the updated values ($q(T_{fire})$, $\dot{q}(T_{fire})$, etc.) are released as representing $q(T)$, $\dot{q}(T)$, etc. In this manner, the RCS jet firing is delayed, but the delay is not allowed to enter the integrals. Cease-firing signals are delayed in the same manner. Obviously, when a jet is calculated to remain on or off during an entire 50 ms. interval, $\Delta\tau = T \doteq 50$ ms. See Figure 1A for the exact values of $\Delta\tau$.